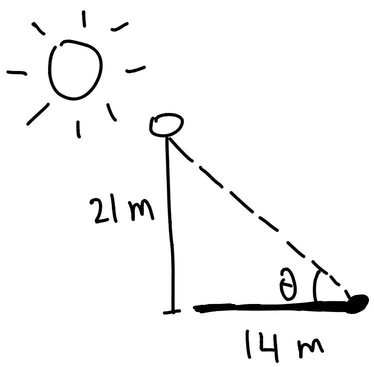
1.4 Worksheet

Name: KEY

1.) Find the angle of elevation of the sun at the moment when a 21 m flag pole casts a 14 m shadow.



$\theta =$ angle of elevation

$\tan\theta = \dfrac{21}{14}$

$\Rightarrow \tan^{-1}(\tan\theta) = \tan^{-1}\left(\frac{21}{14}\right)$

$\Rightarrow \theta = \tan^{-1}\left(\frac{21}{14}\right)$

$= \boxed{56.31^\circ}$

2.) A large airplane (plane A) flying at 31,000 ft sights a smaller plane (plane B) traveling at an altitude of 28,000 ft. The angle of depression is 37°. What is the line-of-sight distance between the two planes?

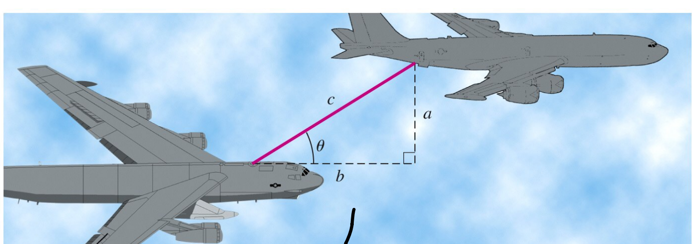
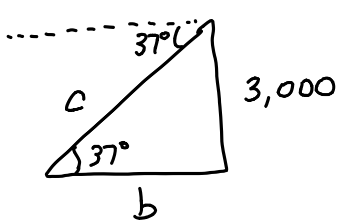


Figure 1.4.3



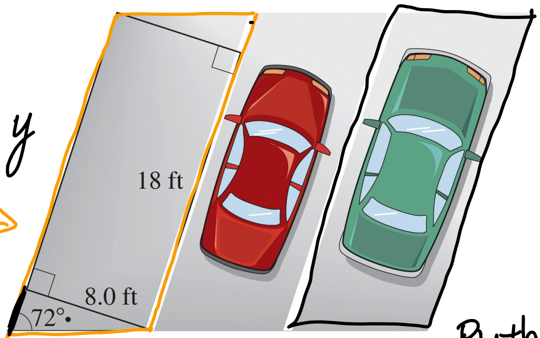
$\sin(37) = \dfrac{3{,}000}{c}$

$c\sin(37) = 3{,}000$

$c = \dfrac{3{,}000}{\sin(37)}$

$\approx \boxed{4985 \text{ ft}}$

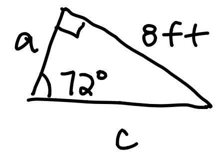
3.) To accommodate cars of most sizes, a parking space needs to contain an 18ft by 8.0 ft rectangle as shown in the figure. If a diagonal parking space makes an angle of 72° with the horizontal, how long are the sides of the parallelogram that contain the rectangle?



y

want to find these two lengths

x



$\tan(72) = \frac{8}{a}$

$\Rightarrow a = \frac{8}{\tan(72)} = 2.6$

Pythagorean Theorem

$2.6^2 + 8^2 = c^2$

$70.757 = c^2$

$8.4 = c$

Trig Ratios

$\sin(72) = \frac{8}{c}$

$\Rightarrow c = \frac{8}{\sin(72)} = 8.4$

so $\boxed{x = 8.4 \text{ ft}}$

$y = 2.6 + 18 = \boxed{20.6 \text{ ft}}$

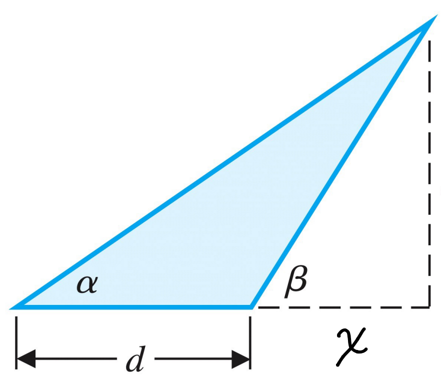
4.) Using the figure, show that $h = \dfrac{d}{\cot\alpha - \cot\beta}$

a. Label the side adjacent to $\angle\beta$ as x

b. Write an equation for the $\cot\alpha$. Write a second equation for the $\cot\beta$.

c. Subtract the second equation from above from the first equation.

d. Simplify to see that $h = \dfrac{d}{\cot\alpha - \cot\beta}$



$\cot\beta =$

$\cot\alpha = \frac{d+x}{h}$

$\cot\beta = \frac{x}{h}$

$\cot\alpha - \cot\beta = \frac{d+x}{h} - \frac{x}{h}$

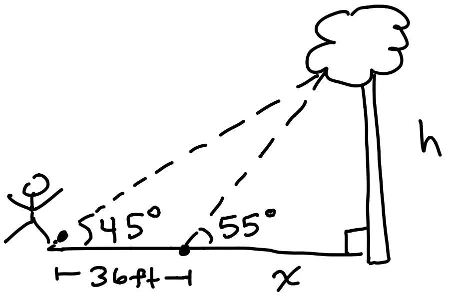
$\cot\alpha - \cot\beta = \frac{d+x-x}{h}$

$\cot\alpha - \cot\beta = \frac{d}{h}$

$h(\cot\alpha - \cot\beta) = d$

$$\boxed{h = \frac{d}{\cot\alpha - \cot\beta}}$$

5.) A surveyor wants to determine the height of a tall tree. He stands at some distance from the tree and determines that the angle of elevation to the top of the tree is 45°. He moves 36 ft closer to the tree, and now the angle of elevation is 55°. How tall is the tree?



$\cot(45) = \frac{36+x}{h}$

$\cot(55) = \frac{x}{h}$

$\cot(45) - \cot(55) = \frac{36+x}{h} - \frac{x}{h}$

$\cot(45) - \cot(55) = \frac{36}{h}$

$\Rightarrow h = \frac{36}{\cot(45) - \cot(55)}$

$= \frac{36}{1 - 0.7}$

$\approx \boxed{120 \text{ ft}}$

6.) An island is 4 mi offshore in a large bay. A water pipeline is to be run from a water tank on the shore to the island, as indicated in the figure below. The pipeline costs \$40,000 per mile in the ocean and \$20,000 per mile on the land.

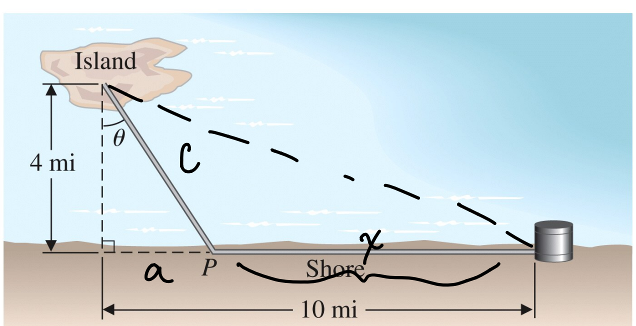
Question: is θ an angle of depression or elevation?

a. Find an expression in terms of θ that represents the distance from the island to the point P on shore. (e.g. $y = 3\csc\gamma$ is an expression in terms of γ)

b. Find an expression in terms of θ that represents the distance from the point P on shore to the water tank.

c. Express the total cost C of the pipeline in terms of θ.

d. Find C for $\theta = 15°$. Round your answer to the nearest hundred dollars.



Exercise 1.4.35

ⓐ $\cos(\theta) = \frac{4}{c} \Rightarrow c\cos(\theta) = 4$

$\Rightarrow c = \frac{4}{\cos\theta}$

ⓑ $\tan\theta = \frac{a}{4} \Rightarrow 4\tan\theta = a$

so $x = 10 - a$

$= 10 - 4\tan\theta$

ⓒ $C = 40{,}000\left(\frac{4}{\cos\theta}\right) + 20{,}000(10 - 4\tan\theta)$

ⓓ $C = 40{,}000\left(\frac{4}{\cos 10}\right) + 20{,}000(10 - 4\tan 10) = 40{,}000(4.062) + 20000(9.295)$

$\approx \boxed{\$348{,}362}$