2.3 Worksheet

Name: KEY

1.) Find the exact value of the six trigonometric functions if the terminal side of the angle x contains the point $P = (-5, -12)$ $= (a, b)$

$r = \sqrt{(-5)^2 + (-12)^2}$
$= \sqrt{25 + 144}$
$= \sqrt{169}$
$= 13$

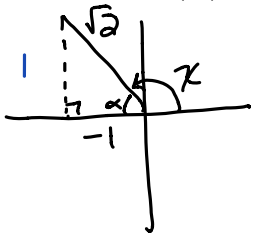
$\sin x = \frac{b}{r} = \frac{-12}{13}$ $\quad \csc x = \frac{r}{b} = \frac{13}{-12}$

$\cos x = \frac{a}{r} = \frac{-5}{13}$ $\quad \sec x = \frac{r}{a} = \frac{13}{-5}$

$\tan x = \frac{b}{a} = \frac{-12}{-5} = \frac{12}{5}$ $\quad \cot x = \frac{a}{b} = \frac{-5}{-12} = \frac{5}{12}$

2.) Find the exact value of the other five trigonometric functions for the angle x (without finding x) for each of the following:

(a.) $\sec x = -\sqrt{2}$; x is a quadrant II angle



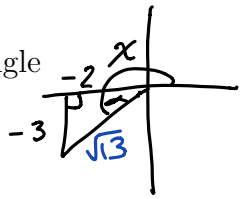
$\frac{1}{\cos x} = \frac{-\sqrt{2}}{1} \Rightarrow \cos x = -\frac{1}{\sqrt{2}}$

$\Rightarrow (-1)^2 + y^2 = (\sqrt{2})^2$
$\Rightarrow 1 + y^2 = 2$
$\Rightarrow y^2 = 1$
$\Rightarrow y = 1$

$\sin x = \frac{1}{\sqrt{2}} = \boxed{\frac{\sqrt{2}}{2}}$ $\quad \cos x = \frac{-1}{\sqrt{2}} = \boxed{\frac{-\sqrt{2}}{2}}$ $\quad \tan x = \frac{1}{-1} = \boxed{-1}$

$\csc x = \frac{\sqrt{2}}{1} = \boxed{\sqrt{2}}$ $\quad \cot x = \frac{-1}{1} = \boxed{-1}$

(b.) $\tan x = \frac{3}{2}$; x is a quadrant III angle



$\sqrt{(-2)^2 + (-3)^2} = \sqrt{4 + 9}$
$r = \sqrt{13}$

$\sin x = \frac{-3}{\sqrt{13}} = \boxed{\frac{-3\sqrt{13}}{13}}$ $\quad \sec x = \boxed{\frac{\sqrt{13}}{-2}}$

$\cos x = \frac{-2}{\sqrt{13}} = \boxed{\frac{-2\sqrt{13}}{13}}$ $\quad \cot x = \boxed{\frac{2}{3}}$

$\csc x = \boxed{\frac{\sqrt{13}}{-3}}$

3.) A point (x, y) lies on a circle if it satisfies the equation $x^2 + y^2 = r^2$. Check to see if the following points lie on the given circles. If it does not lie on the given circle, write the equation of the circle it does lie on.

(a.) $P = (3, 4)$; $\quad x^2 + y^2 = 36$

$(3)^2 + (4)^2 = 9 + 16 = 25 \neq 36$

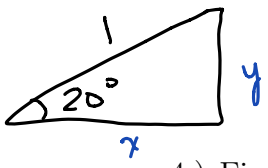
So $(3,4)$ does <u>not</u> lie on the circle $x^2 + y^2 = 36$

Since $3^2 + 4^2 = 25$ the point $(3,4)$ is on $x^2 + y^2 = 25$

(b.) $P = \left(\frac{3}{5}, \frac{4}{5}\right)$; $\quad x^2 + y^2 = 1$

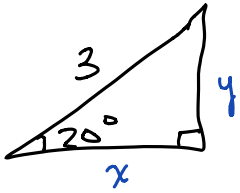
$\left(\frac{3}{5}\right)^2 + \left(\frac{4}{5}\right)^2 = \frac{9}{25} + \frac{16}{25} = \frac{25}{25} = 1$

So P is on the given circle



$\cos(20) = \frac{x}{1} \Rightarrow x = \cos(20) \approx 0.9397$

$\sin(20) = \frac{y}{1} \Rightarrow y = \sin(20) \approx 0.342$



$\cos(20) = \frac{x}{3} \Rightarrow x = 3\cos(20) \approx 2.819$

$\sin(20) = \frac{y}{3} \Rightarrow y = 3\sin(20) \approx 1.026$

4.) Find the six trig functions of $\theta = 20°$ if the terminal side of θ in standard position contains a point on the circle $x^2 + y^2 = 9$? if the terminal side contains a point on the circle $x^2 + y^2 = 1$?

$x^2 + y^2 = 1$

$\sin 20 = 0.342$

$\cos 20 = 0.9397$

$\tan 20 = \frac{0.342}{0.9397} = 0.364$

$\csc 20 = \frac{1}{0.342} = 2.924$

$\sec 20 = \frac{1}{0.9397} = 1.064$

$\cot 20 = \frac{1}{0.364} = 2.747$

$x^2 + y^2 = 30$

$\sin 20 = \frac{1.026}{3} = 0.342$

$\cos 20 = \frac{2.819}{3} = 0.9397$

$\tan 20 = \frac{0.342}{0.9397} = 0.364$

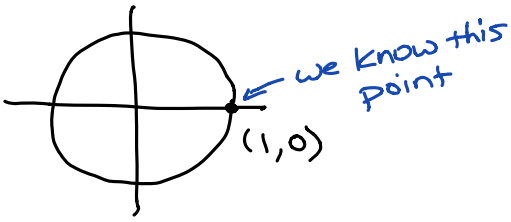
$\csc 20 = 2.924$

$\sec 20 = 1.064$

$\cot 20 = 2.747$

5.) Find sine, cosine, and tangent of the following angles if the terminal side of the angle in standard position contains a point on the circle $x^2 + y^2 = 1$. (*Hint: convert the angles to degrees and then use special right triangles.*)

(a.) $\theta = 0$



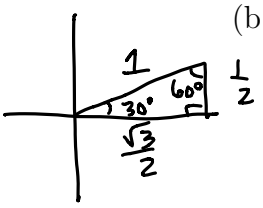
$r = 1 \Rightarrow$

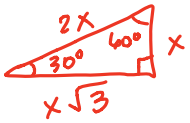
$\sin 0 = \boxed{0}$

$\cos 0 = \boxed{1}$

$\tan 0 = \frac{0}{1} = \boxed{1}$

(b.) $\theta = \frac{\pi}{6}$





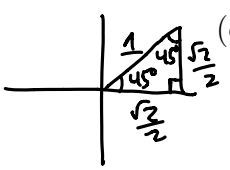
$2x = 1 \Rightarrow x = \frac{1}{2}$

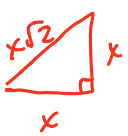
$\sin\frac{\pi}{6} = \frac{\left(\frac{1}{2}\right)}{1} = \boxed{\frac{1}{2}}$

$\cos\frac{\pi}{6} = \frac{\left(\frac{\sqrt{3}}{2}\right)}{1} = \boxed{\frac{\sqrt{3}}{2}}$

$\tan\frac{\pi}{6} = \frac{\left(\frac{1}{2}\right)}{\left(\frac{\sqrt{3}}{2}\right)} = \frac{1}{2}\cdot\frac{2}{\sqrt{3}} = \frac{1}{\sqrt{3}} = \boxed{\frac{\sqrt{3}}{3}}$

(c.) $\theta = \frac{\pi}{4}$





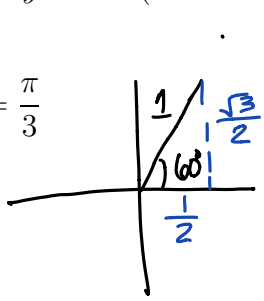
$x\sqrt{2} = 1 \Rightarrow x = \frac{1}{\sqrt{2}} = \frac{\sqrt{2}}{2}$

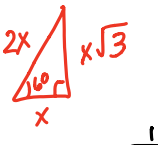
$\sin\frac{\pi}{4} = \frac{\left(\frac{\sqrt{2}}{2}\right)}{1} = \boxed{\frac{\sqrt{2}}{2}}$

$\cos\frac{\pi}{4} = \frac{\left(\frac{\sqrt{2}}{2}\right)}{1} = \boxed{\frac{\sqrt{2}}{2}}$

$\tan\frac{\pi}{4} = \frac{\left(\frac{\sqrt{2}}{2}\right)}{\left(\frac{\sqrt{2}}{2}\right)} = \boxed{1}$

(d.) $\theta = \frac{\pi}{3}$





$2x = 1 \Rightarrow x = \frac{1}{2}$

$\frac{1}{2}\cdot\frac{\sqrt{3}}{1} = \frac{\sqrt{3}}{2}$

$\sin\frac{\pi}{3} = \frac{\left(\frac{\sqrt{3}}{2}\right)}{1} = \boxed{\frac{\sqrt{3}}{2}}$

$\cos\frac{\pi}{3} = \frac{\left(\frac{1}{2}\right)}{1} = \boxed{\frac{1}{2}}$

$\tan\frac{\pi}{3} = \frac{\left(\frac{\sqrt{3}}{2}\right)}{\left(\frac{1}{2}\right)} = \frac{\sqrt{3}}{2}\cdot\frac{2}{1} = \boxed{\sqrt{3}}$

(e.) $\theta = \frac{\pi}{2}$

$\sin\frac{\pi}{2} = \boxed{1}$

$\cos\frac{\pi}{2} = \boxed{0}$

$\tan\frac{\pi}{2} = \frac{1}{0} = \boxed{\text{undefined}}$