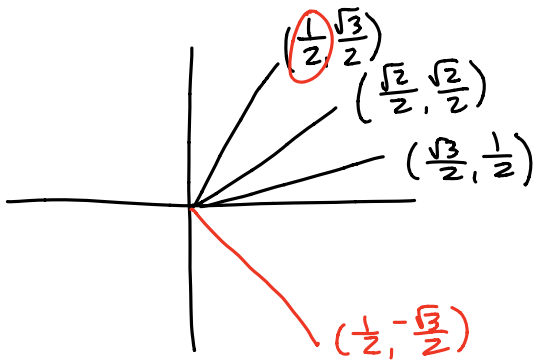


## 5.3 Worksheet

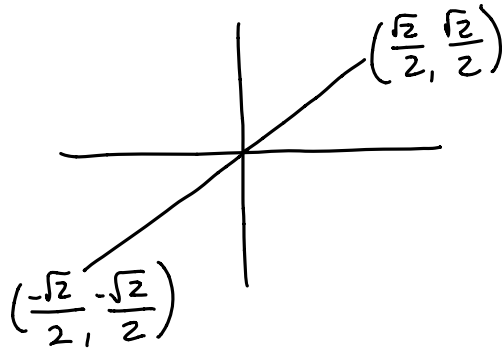
1. Find the exact solution(s) to the following for all  $x$ .

(a.)  $\cos x = \frac{1}{2}$



$$\boxed{x = \frac{\pi}{3} + 2k\pi, \frac{5\pi}{3} + 2k\pi}$$

(b.)  $\tan x = 1$



$x = 45^\circ, 215^\circ$  for  $0 \leq x < 2\pi$

in general,  $\boxed{x = 45^\circ + k/80^\circ}$

2. Find the exact solution(s) to the following for  $0^\circ \leq \theta < 360^\circ$ .

(a.)  $2\sin\theta - \sqrt{3} = 0$   
 $\quad \quad \quad +\sqrt{3} \quad +\sqrt{3}$

$$2\sin\theta = \sqrt{3}$$

$$\sin\theta = \frac{\sqrt{3}}{2}$$

$$\boxed{\theta = 60^\circ, 120^\circ}$$

(b.)  $2\sin\theta + 1 = 0$   
 $\quad \quad \quad -1 \quad -1$

$$2\sin\theta = -1$$

$$\sin\theta = -\frac{1}{2}$$

$$\boxed{\theta = 210^\circ, 330^\circ}$$

3. Find the exact solution(s) to the following for  $0 \leq x < 2\pi$ .

(a.)  $\cos x = \cot x$

$$\cos x = \frac{\cos x}{\sin x}$$

$$\cos x \sin x = \cos x$$

$$\cos x \sin x - \cos x = 0$$

$$\cos x (\sin x - 1) = 0$$

$$\Rightarrow \cos x = 0, \sin x - 1 = 0$$

$$x = \frac{\pi}{2}, \frac{3\pi}{2}$$

$$\sin x = 1$$

$$x = \frac{\pi}{2}$$

$$\boxed{x = \frac{\pi}{2}, \frac{3\pi}{2}}$$

(b.)  $2 \cos^2 x - \cos x = 0$

$$\cos x (2 \cos x - 1) = 0$$

$$\Rightarrow \cos x = 0, 2 \cos x - 1 = 0$$

$$x = \frac{\pi}{2}, \frac{3\pi}{2}$$

$$2 \cos x = 1$$

$$\cos x = \frac{1}{2}$$

$$x = \frac{\pi}{3}, \frac{5\pi}{3}$$

$$\boxed{x = \frac{\pi}{2}, \frac{\pi}{3}, \frac{3\pi}{2}, \frac{5\pi}{3}}$$

4. Find the exact solution(s) to the following for all  $x$ .

(a.)  $\cos^2 x + 4 \sin x = -4$

$$1 - \sin^2 x + 4 \sin x = -4$$

$$1 - \sin^2 x + 4 \sin x + 4 = 0$$

$$\underbrace{-\sin^2 x}_{"y^2"} + \underbrace{4 \sin x}_{"y"} + 5 = 0$$

$$\Rightarrow -y^2 + 4y + 5 = 0$$

$$y^2 - 4y - 5 = 0$$

$$(y-5)(y+1) = 0$$

$$y = 5 \quad y = -1$$

$$\Rightarrow \sin x = 5$$

$$x = \sin^{-1}(5) \quad \boxed{DNE}$$

$$\Rightarrow \sin x = -1$$

$$\boxed{x = \frac{3\pi}{2} + 2k\pi}$$

(b.)  $10 \sin^2 x + 7 \sin x = 6$

$$10 \underbrace{\sin^2 x}_{"y^2"} + 7 \underbrace{\sin x}_{"y"} - 6 = 0$$

$$10y^2 + 7y - 6 = 0$$

$$(10y^2 - 5y) + (12y - 6) = 0$$

$$5y(2y-1) + 6(2y-1) = 0$$

$$(2y-1)(5y+6) = 0$$

$$y = \frac{1}{2} \quad y = -\frac{6}{5}$$

$$\Rightarrow \sin x = \frac{-6}{5} \Rightarrow x = \sin^{-1}\left(-\frac{6}{5}\right) \quad \boxed{DNE}$$

5. Find the exact solution(s) to  $\cos^2 \theta = \frac{1}{2} \sin 2\theta$  for  $0^\circ \leq x < 360^\circ$ .

$$\cos^2 \theta = \frac{1}{2} (2 \sin \theta \cos \theta)$$

$$\cos^2 \theta = \sin \theta \cos \theta$$

$$\cos^2 \theta - \sin \theta \cos \theta = 0$$

$$\cos \theta (\cos \theta - \sin \theta) = 0$$

$$\Rightarrow \cos \theta = 0, \cos \theta - \sin \theta = 0$$

$$\cos \theta = \sin \theta$$

$$\Rightarrow \cos \theta = 0$$

$$\theta = 90^\circ, 270^\circ$$

$$\Rightarrow \cos \theta = \sin \theta$$

$$\theta = 45^\circ, 215^\circ$$

$$\Rightarrow \sin x = \frac{1}{2}$$

$$\boxed{x = \frac{\pi}{3} + 2k\pi, \frac{2\pi}{3} + 2k\pi}$$

$$\text{so } \boxed{\theta = 45^\circ, 90^\circ, 215^\circ, 270^\circ}$$