Reciprocal Identities

100 Q) $\csc x = ?$

A) $\boxed{\frac{1}{\sin x}}$

200 Q) $\sin x \sec x = ?$

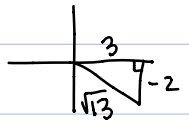
A) $\sin x \sec x = \sin x \cdot \frac{1}{\cos x} = \frac{\sin x}{\cos x} = \boxed{\tan x}$

300 Q) $\frac{\sec x}{\tan x} = ?$

A) $\frac{\sec x}{\tan x} = \frac{\left(\frac{1}{\cos x}\right)}{\left(\frac{\sin x}{\cos x}\right)} = \frac{1}{\cos x} \cdot \frac{\cos x}{\sin x} = \frac{1}{\sin x} = \boxed{\csc x}$

400 Q) If $\cot x = \frac{-3}{2}$ and $\cos x > 0$ find the exact value of the remaining trig functions.

A)



$\sin x = \frac{-2}{\sqrt{13}} = \frac{-2\sqrt{13}}{3}$ $\quad \csc x = -\frac{\sqrt{13}}{2}$

$\cos x = \frac{3}{\sqrt{13}} = \frac{3\sqrt{13}}{13}$ $\quad \sec x = \frac{\sqrt{13}}{3}$

$\tan x = \frac{-2}{3}$ $\quad \cot x = \frac{-3}{2}$

Whoops! I did this completely wrong in class!

500 (Q) If $\cos x = \frac{-4}{5}$ and $\tan x = \frac{3}{4}$ find the exact values of the remaining trig functions.

$$\tan x = \frac{\sin x}{\cos x} \Rightarrow \tan x \cos x = \sin x$$
$$\Rightarrow \left(\frac{3}{4}\right)\left(\frac{-4}{5}\right) = \frac{-3}{5} = \sin x$$

$\sin x = \frac{-3}{5}$	$\csc x = \frac{-5}{3}$
$\cos x = \frac{-4}{5}$	$\sec x = \frac{-5}{4}$
$\tan x = \frac{3}{4}$	$\cot x = \frac{4}{3}$

Pythagorean Identities

100 (Q) This is the standard Pythagorean Identity

A) $\boxed{\sin^2 x + \cos^2 x = 1}$

200 Q) Prove that $\tan^2 x + 1 = \sec^2 x$.

A.) $\sin^2 x + \cos^2 x = 1 \Rightarrow \frac{\sin^2 x}{\cos^2 x} + \frac{\cos^2 x}{\cos^2 x} = \frac{1}{\cos^2 x}$

$$\Rightarrow \boxed{\tan^2 x + 1 = \sec^2 x}$$

300 Q) simplify $\dfrac{1-(\cos x-\sin x)^2}{\cos x}$

A) $\dfrac{1-(\cos x-\sin x)^2}{\cos x} = \dfrac{1-(\cos x-\sin x)(\cos x-\sin x)}{\cos x}$

$= \dfrac{1-[\cos^2 x-2\cos x\sin x+\sin^2 x]}{\cos x}$

$= \dfrac{1-[-2\cos x\sin x+(\sin^2 x+\cos^2 x)}{\cos x}$

$= \dfrac{1+2\cos x\sin x-1}{\cos x}$

$= \dfrac{2\cos x\sin x}{\cos x}$

$= \boxed{2\sin x}$

400 Q) simplify $\dfrac{1}{\csc^2 x}+\dfrac{1}{\sec^2 x}$

A) $\dfrac{1}{\csc^2 x}+\dfrac{1}{\sec^2 x} = \sin^2 x+\cos^2 x = \boxed{1}$

500 Q) show $\dfrac{\sin^2 x}{1-\cos x} = 1+\cos x$

A) $\dfrac{\sin^2 x}{1-\cos x} = \dfrac{1-\cos^2 x}{1-\cos x} = \dfrac{(1+\cos x)(1-\cos x)}{1-\cos x} = \boxed{1+\cos x}$

Negative Identities

100 Q) $\cos(-x) = ?$

A) $\cos x$

200 Q) $\csc(-x) = ?$

A) $\csc(-x) = \frac{1}{\sin(-x)} = \frac{1}{-\sin x} = \boxed{-\csc x}$

300 Q) $\sec(-x) = ?$

A) $\sec(-x) = \frac{1}{\cos(-x)} = \frac{1}{\cos x} = \boxed{\sec x}$

400 Q) Simplify $\sin x - \frac{\tan(-x)}{\sec x}$

A) $\sin x - \frac{\tan(-x)}{\sec x} = \sin x - \frac{\frac{\sin(-x)}{\cos(-x)}}{\left(\frac{1}{\cos x}\right)}$

$= \sin x - \frac{\sin(-x)}{\cos(-x)} \cdot \frac{\cos x}{1}$

$= \sin x - \frac{(-\sin x)}{\cos x} \cdot \frac{\cos x}{1}$

$= \sin x + \sin x$

$= \boxed{2\sin x}$

500 Q) simplify $\cot(-x)$ completely.

A) $\cot(-x) = \frac{\cos(-x)}{\sin(-x)} = \frac{\cos x}{-\sin x} = \boxed{-\cot x}$

Verifying Identities

100 Q) $\frac{\sin x}{\tan x} = \cos x$

A) $\frac{\sin x}{\tan x} = \frac{\sin x}{\left(\frac{\sin x}{\cos x}\right)} = \frac{\sin x}{1} \cdot \frac{\cos x}{\sin x} = \boxed{\cos x}$

200 Q) $\frac{1}{\cos^2 x} - 1 = \tan^2 x$

A) $\frac{1}{\cos^2 x} - 1 = \sec^2 x - 1 = \boxed{\tan^2 x}$

300 Q) Verify $\sec x(\sin x - \cos x) = \tan x - 1$

A) $\sec x(\sin x - \cos x) = \frac{1}{\cos x}(\sin x - \cos x)$

$= \frac{\sin x}{\cos x} - \frac{\cos x}{\cos x}$

$= \boxed{\tan x - 1}$

400 Q) verify $\frac{1+\cos x}{\sin x} + \frac{\sin x}{1+\cos x} = 2\csc x$

A) $\frac{1+\cos x}{\sin x}\cdot\frac{(1+\cos x)}{(1+\cos x)} + \frac{\sin x}{1+\cos x}\cdot\frac{(\sin x)}{(\sin x)}$

$= \frac{(1+\cos x)(1+\cos x)}{\sin x(1+\cos x)} + \frac{\sin^2 x}{\sin x(1+\cos x)}$

$= \frac{1+2\cos x+\cos^2 x+\sin^2 x}{\sin x(1+\cos x)}$

$= \frac{1+2\cos x+1}{\sin x(1+\cos x)}$

$= \frac{2+2\cos x}{\sin x(1+\cos x)}$

$= \frac{2(1+\cos x)}{\sin x(1+\cos x)}$

$= \frac{2}{\sin x}$

$= 2\cdot\frac{1}{\sin x}$

$= \boxed{2\csc x}$

500 Q) verify $\frac{1-\cos^2 x}{\sin^3 x} = \csc x$

A) $\frac{1-\cos^2 x}{\sin^3 x} = \frac{\sin^2 x}{\sin^3 x} = \frac{1}{\sin x} = \boxed{\csc x}$

Remember me?

100 Q) What is the period of $y = A\tan(BX+C)$

A) period: $\boxed{\frac{\pi}{B}}$

200 Q) Where does the graph of $\tan x$ have vertical asymptotes?

A) vertical asymptotes occur at $\boxed{x = \frac{\pi}{2} + k\pi}$

300 Q) List the trig functions that have a period of 2π

A) $\sin x, \cos x, \csc x, \sec x$

400 Q) Find the phase shift and amplitude of $y = 5\csc(4x-1)$

A) phase shift: $\frac{-(-1)}{4} = \boxed{\frac{1}{4}}$

$\csc x$ has no amplitude

500 Q) Find max and min of $y = -13 + 4\sin(2x)$

A) $-1 \leq \sin x \leq 1$

$-1 \leq \sin(2x) \leq 1$

$-4 \leq 4\sin(2x) \leq 4$

$-17 \leq -13 + 4\sin(2x) \leq -9$

$\boxed{\begin{array}{l}\text{Max: } -9 \\ \text{Min: } -17\end{array}}$