Midterm 3 (100 pts.)　　　　Name: KEY

True or False (3 pts. each)

Answer the following by circling TRUE or FALSE. If the answer is false you must explain why in the space provided for full credit.

1.) T F ______

$\mathbf{u} = < -3, 4 >$ and $\mathbf{v} = < 4, -3 >$ are orthogonal vectors

$u \cdot v = (-3)(4) + (4)(-3) = -24$

2.) T F $\csc^{-1}(x) = \sin^{-1}\left(\frac{1}{x}\right)$

$\csc^{-1} x = \dfrac{1}{\sin^{-1} x}$

3.) T F ______

The domain of $\sin^{-1} x$ is $[-1, 1]$

4.) T F $\frac{1}{2}(15)(10) = \frac{150}{2}\ cm^2$

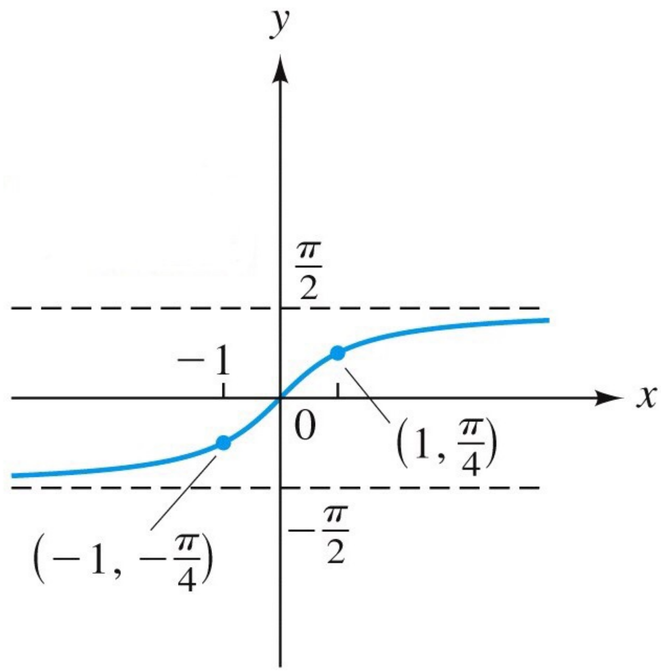
Given a triangle with base $b = 15$ cm and height $h = 10$ cm the area $A = 150\ cm^2$

5.) T F ______

The law of cosines can be used to solve a triangle given $a = 6$ cm, $b = 8$ cm, and $c = 12$ cm.

Multiple Choice (5 pts. each)

6.) Which equation best describes the graph?



(a) $y = \sin^{-1} x$

(b) $y = \cos^{-1} x$

(c) $y = \tan^{-1} x$

(d) $y = \csc^{-1} x$

(e) $y = \sec^{-1} x$

(f) $y = \cot^{-1} x$

7.) Which of the following triangles can be solved using Law of Sines?

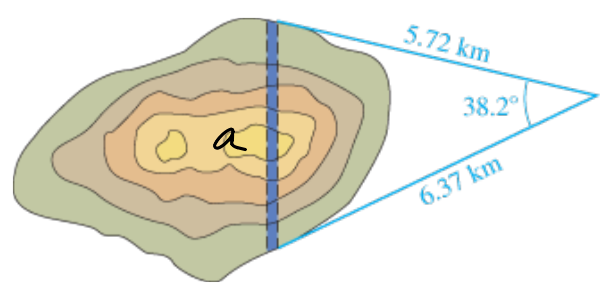
(a) $a = 38$ mi, $b = 73$ mi, $c = 69$ mi   SSS X

(b) $a = 71$ cm, $b = 85$ cm, $\gamma = 124°$ SSA (but γ not opp a or b) X

(c) $\alpha = 83°$, $\beta = 65°$, and $c = 56$ km   AAS

(d) $\beta = 57°$, $\gamma = 126°$ and $a = 60$ ft   AAS $(57°+126° = 183° > 180°)$

8.) A tunnel for a highway is to be constructed through a mountain, as indicated in the figure below. How long is the tunnel?



(a) 0.65 km

(b) 4.00 km

(c) 5.31 km

(d) 16.03 km

$$a^2 = (6.37)^2 + (5.72)^2 - 2(5.72)(6.37)\cos(38.2°)$$
$$= 73.2953 - 72.8728\cos(38.2°)$$
$$= 16.0277$$
$$\Rightarrow a = \sqrt{16.0277} = \boxed{4.0035}$$

9.) The graph below is one-to-one on all of the following intervals except:

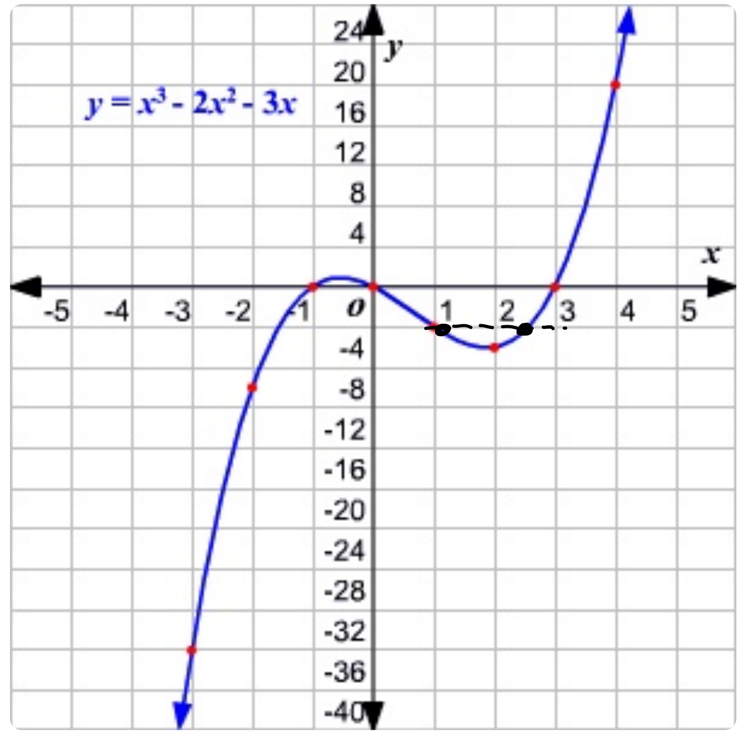
(a) $[-3, -1]$ ✓

(b) $[0, 2]$ ✓

(c) $[1, 2]$ ✓

(d) $[1, 3]$ X

(e) $[2, 4]$ ✓



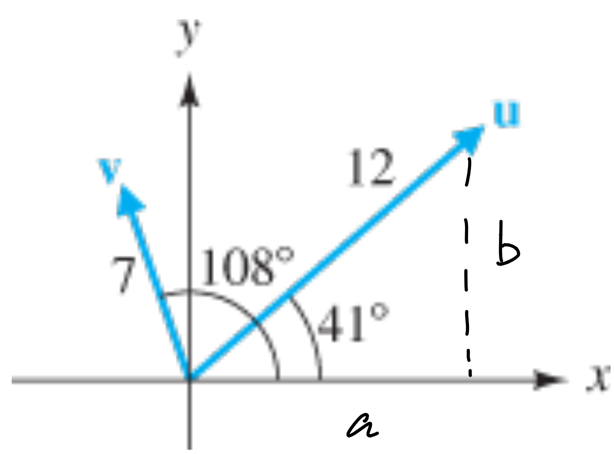
10.) Find the scalar components a and b of vector the vector $\mathbf{u} = < a, b >$, given the diagram below.

(a) $a = 12\sin(41°)$, $b = 12\cos(41°)$

(b) $a = 12\cos(41°)$, $b = 12\sin(41°)$

(c) $a = 7\sin(67°)$, $b = 7\cos(67°)$

(d) $a = 7\cos(67°)$, $b = 7\sin(67°)$



$\cos(41°) = \frac{a}{12} \Rightarrow \boxed{a = 12\cos(41°)}$

$\sin(41°) = \frac{b}{12} \Rightarrow \boxed{b = 12\sin(41°)}$

Short Answer (10 pts. each)

11.) Let $A = (-2, 3)$, $B = (5, 7)$, $C = (-2, 0)$, and $D = (8, 0)$.

(a) (1.5 pts.) Represent the geometric vector $\overrightarrow{AB}$ as a standard vector $\mathbf{v} = < a, b >$.

$\overrightarrow{AB} = \langle 5-(-2), 7-3 \rangle = \boxed{\langle 7, 4 \rangle}$

(b) (1.5 pts.) Represent the geometric vector $\overrightarrow{CD}$ as a standard vector using i and j components.

$\overrightarrow{CD} = \langle 8-(-2), 0-0 \rangle = \langle 10, 0 \rangle = 10i + 0j = \boxed{10i}$

(c) (4 pts.) Find the angle θ between $\overrightarrow{AB}$ and $\overrightarrow{CD}$. *Round to the nearest degree.*

$\overrightarrow{AB} = \langle 7, 4 \rangle \quad \overrightarrow{CD} = \langle 10, 0 \rangle$

$|\overrightarrow{AB}| = \sqrt{7^2+4^2} = \sqrt{49+16} = \sqrt{65}$

$|\overrightarrow{CD}| = \sqrt{10^2+0^2} = \sqrt{100} = 10$

$\overrightarrow{AB} \cdot \overrightarrow{CD} = (7)(10) + (4)(0) = 70$

$\cos\theta = \dfrac{70}{(\sqrt{65})(10)} = \dfrac{7}{\sqrt{65}}$

$\Rightarrow \theta = \cos^{-1}\left(\dfrac{7}{\sqrt{65}}\right) = \boxed{29.74^\circ}$

(d) (3 pts.) Let $\mathbf{u} = \overrightarrow{AB}$ and $\mathbf{v} = \overrightarrow{CD}$. Find the $Comp_{\mathbf{v}}\mathbf{u}$.

$Comp_v u = |u| \cos\theta$

$= (\sqrt{65}) \cdot \dfrac{7}{\sqrt{65}}$

$= \boxed{7}$

12.) Let $\mathbf{u} = <0, -11>$, $\mathbf{v} = <-2, 8>$, and $\mathbf{w} = <4, -4>$.

(a) (1.5 pts.) Find **v+w**

$$v+w = \langle -2+4, 8-4 \rangle = \boxed{\langle 2, 4 \rangle}$$

(b) (1.5 pts.) Find **5u+2v-3w**

$$5u = 5\langle 0, -11 \rangle = \langle 0, -55 \rangle$$

$$2v = 2\langle -2, 8 \rangle = \langle -4, 16 \rangle$$

$$-3w = -3\langle 4, -4 \rangle = \langle -12, 12 \rangle$$

$$\Rightarrow \quad 5u+2v-3w = \langle 0-4-12, -55+16+12 \rangle = \boxed{\langle -16, -27 \rangle}$$

(c) (3 pts.) Find a unit vector **q** in the same direction as **w**

$$|w| = \sqrt{(4)^2+(-4)^2} = \sqrt{16+16} = \sqrt{32}$$

$$q = \frac{1}{\sqrt{32}}\langle 4, -4 \rangle = \boxed{\left\langle \frac{4}{\sqrt{32}}, \frac{-4}{\sqrt{32}} \right\rangle}$$

(d) (4 pts.) Find a vector **p** of length 3 in the direction opposite **v**

$$|v| = \sqrt{(-2)^2+(8)^2} = \sqrt{4+64} = \sqrt{68}$$

$$\frac{1}{\sqrt{68}}\langle -2, 8 \rangle = \left\langle \frac{-2}{\sqrt{68}}, \frac{8}{\sqrt{68}} \right\rangle$$ (unit vector in direction of v)

$$\left\langle \frac{2}{\sqrt{68}}, \frac{-8}{\sqrt{68}} \right\rangle$$ (unit vector in direction opp v)

$$\boxed{p = \left\langle \frac{6}{\sqrt{68}}, \frac{-24}{\sqrt{68}} \right\rangle}$$ vector of length 3 in direction opp v

13.) Find the exact solution(s) to the following for all x.

(a) (6 pts.) $2\cos^2\theta + 3\cos\theta = -1$

$2\cos^2\theta + 3\cos\theta + 1 = 0$ let $y = \cos\theta$

$2y^2 + 3y + 1 = 0$

$(2y+1)(y+1) = 0$

$\Rightarrow 2y+1=0 \qquad y+1=0$

$2y=-1 \qquad y=-1$

$y = -\frac{1}{2} \qquad \Rightarrow \cos\theta = -1$

$\Rightarrow \cos\theta = -\frac{1}{2} \qquad \Rightarrow \theta = \pi + 2k\pi$

$\Rightarrow \theta = \frac{2\pi}{3} + 2\pi k$

$$\boxed{\theta = \frac{2\pi}{3} + 2k\pi,\ \pi + 2k\pi}$$

(b) (4 pts.) $\theta = \csc^{-1}\left(-\frac{2}{\sqrt{3}}\right)$

$\theta = \csc^{-1}\left(\frac{-2}{\sqrt{3}}\right)$

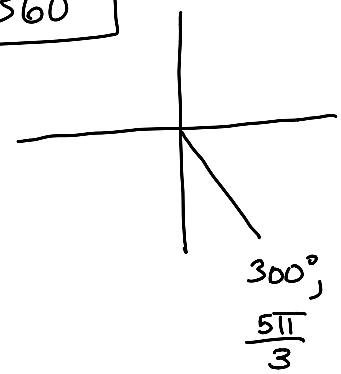
$= \sin^{-1}\left(\frac{1}{\left(\frac{-2}{\sqrt{3}}\right)}\right)$

$= \sin^{-1}\left(\frac{-\sqrt{3}}{2}\right)$

$\Rightarrow \boxed{\theta = 300^\circ + k360^\circ}$

or

$\boxed{\theta = \frac{5\pi}{3} + 2k\pi}$



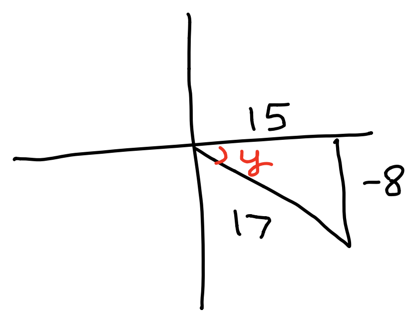
14.) Find the exact value of $\sin\left(2\sin^{-1}\left(-\frac{8}{17}\right)\right)$. $\sin 2x = 2\sin x\cos x$

$\underbrace{\sin^{-1}\left(-\frac{8}{17}\right)}_{x}$

$= 2\,\cancel{\sin}\left(\cancel{\sin^{-1}}\left(\frac{-8}{17}\right)\right)\cos\left(\underbrace{\sin^{-1}\left(\frac{-8}{17}\right)}_{y}\right)$

$= \frac{2}{1}\left(\frac{-8}{17}\right)\left(\frac{15}{17}\right)$

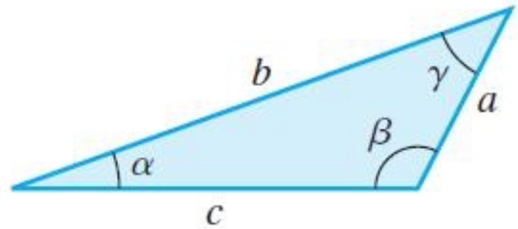
$= \boxed{\frac{-240}{289}}$



$\Rightarrow \cos y = \frac{15}{17}$

15.) Consider the triangular plot below with $b = 18m$, $\gamma = 49°$, and $\beta = 111°$

(a) (6 pts.) Solve the triangle by finding α, a, and c.



$$\alpha = 180° - (\gamma + \beta)$$
$$= 180° - (49° + 111°)$$
$$= 180 - (160°)$$
$$\boxed{\alpha = 20°}$$

$$\frac{\sin\alpha}{a} = \frac{\sin\beta}{b}$$

$$\frac{\sin(20°)}{a} = \frac{\sin(111°)}{18}$$

$$a = \frac{18\sin(20°)}{\sin(111°)}$$

$$\boxed{a = 6.594\, m}$$

$$\frac{\sin\gamma}{c} = \frac{\sin\beta}{b}$$

$$\frac{\sin(49°)}{c} = \frac{\sin(111°)}{18}$$

$$c = \frac{18\sin(49°)}{\sin(111°)}$$

$$\boxed{c = 14.551\, m}$$

(b) (3 pts.) Find the area of the triangle.

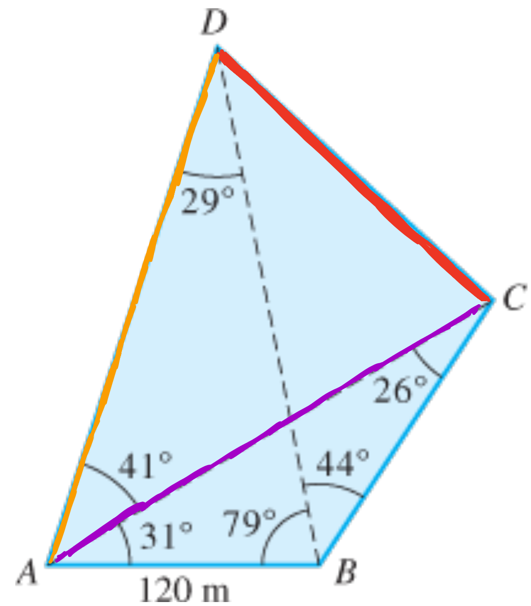
$$A = \frac{1}{2}(14.551)(18)\sin(20)$$
$$= \boxed{44.79\, m^2}$$

meter

(c) (1 pt.)If the cost per square ~~foot~~ is \$4.00, how much will be required to purchase the triangular plot?

$$\text{Cost} = (\$4.00)(44.7906) = \boxed{\$179.16}$$

16.) A plot of land was surveyed with resulting information shown in the figure below. Find the length of CD.



Strategy

1) Find AD
2) Find AC

use Law of Sines

3) Find CD use Law of Cosines

①

$$\frac{\sin(79°)}{AD} = \frac{\sin(29°)}{120}$$

$$\Rightarrow AD = \frac{120\sin(79°)}{\sin(29°)} = 242.972$$

②

$$\frac{\sin(26°)}{120} = \frac{\sin(79°+44°)}{AC}$$

$$\Rightarrow AC = \frac{120\sin(123°)}{\sin(26°)} = 229.578$$

③

$$(CD)^2 = (AD)^2+(AC)^2-2(AD)(AC)\cos(\angle DAC)$$

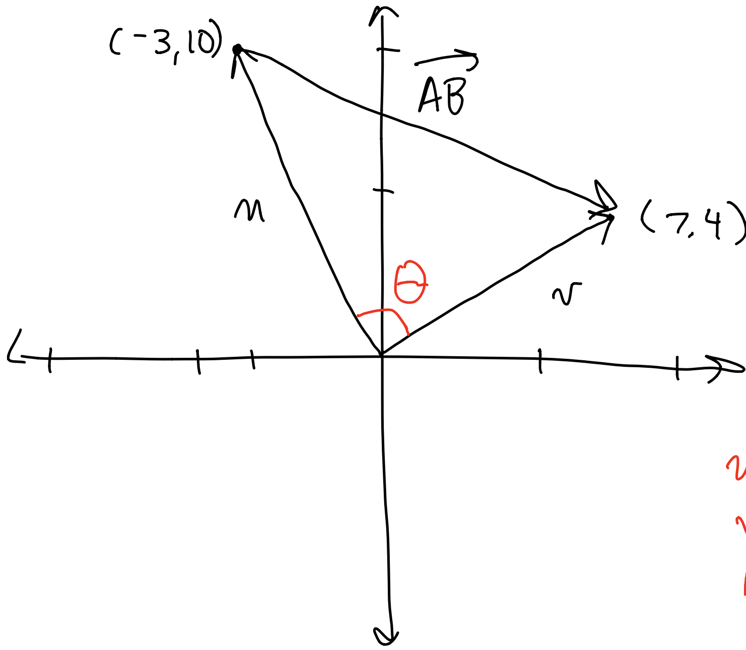
$$= (242.972)^2+(229.578)^2-2(242.972)(229.578)\cos(41°)$$

$$= 111741.4509 - 111562.0516\cos(41°)$$

$$= 27544.5017$$

$$CD = \sqrt{27544.5017} = \boxed{165.9654 \text{ m}}$$

Bonus



$u = -3i + 10j$
$v = 7i + 4j$
$A = (-3, 10)$
$B = (7, 4)$

Find area of the figure with sides formed by u, v, and the geometric vector $\overrightarrow{AB}$

$$|u| = \sqrt{(-3)^2 + (10)^2} = \sqrt{9 + 100} = \sqrt{109}$$

$$|v| = \sqrt{(7)^2 + (4)^2} = \sqrt{49 + 16} = \sqrt{65}$$

$$\cos\theta = \frac{u \cdot v}{|u||v|} = \frac{(-3)(7) + (10)(4)}{(\sqrt{109})(\sqrt{65})} = \frac{-21 + 40}{\sqrt{7085}} = \frac{19}{\sqrt{7085}}$$

$$\Rightarrow \theta = \cos^{-1}\left(\frac{19}{\sqrt{7085}}\right) = 76.954$$

$$A = \frac{1}{2}(\sqrt{109})(\sqrt{65})\sin(76.954) = 40.9999 \approx \boxed{41 \text{ square units}}$$