Project Proposal

Ranthony A.C. Edmonds
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Dr. Steve Alessi

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Proposed Course

Course Title: An Introduction to Modern Algebra Part 1a: Arithmetic, Congruence, and Rings

Motivation: Modern Algebra is usually a year long sequence roughly split into two topics, groups and rings. In general, a group is an algebraic structure satisfying certain properties coupled with an operation. A ring is another type of algebraic structure satisfying certain properties coupled with a binary operation. In a typical sequence, groups are introduced in the first semester and then rings are introduced in the second with the connection being that every ring is a certain type of group but with additional structure. For example, a set of elements could form a group under the operation of addition, while that same set of elements could also form a ring with a binary operation of both addition and multiplication. In this way most courses seek to introduce students to groups and then later to rings by simply adding a bit more structure to the algebraic properties the student has become familiar with.

While I agree with this method in general, and would employ it in a graduate level course for consistency, for an introductory course intended as a first course in abstract algebra I propose

that things should be taught the other way around. Starting off with rings allows for a good treatment of arithmetic, division, prime numbers in a more sophisticated setting. Congruence and modular arithmetic provides intuition for classic rings like $\mathbb{Z}_n$ for a natural number n before a ring's structure is fully understood. When rings are introduced as a mathematical object there is still a lot of intuition present as students see that the number systems they are already familiar with, for example the integers, real numbers, and the complex numbers are in fact rings. There is a textbook by Thomas W. Hungerford that follows this outline of thinking. I used this as an undergraduate student learning these topics for the first time and will use the first three chapters as the primary text for this particular course.

This course would be part of a four part series, Introduction to Modern Algebra Part 1a, 1b, 2a, and 2b with the intention of each being an 8 week course. I think splitting it up this way allows for a more thorough treatment of the subjects in each course and more flexibility for students in when they elect to take it. Many four year colleges only offer modern algebra once every two years and having four 8-week online modules would be helpful for students who want to take this class during the year it is not being offered. What I propose is to make the first part of these sequences as my course project in this class.

Course Objectives

The course has the following learning objectives:

Mathematical Reasoning

1. Students can make sound arguments and write effective proofs based on careful mathematical reasoning

2. Students can effectively communicate mathematics using correct language and notation

3. Students understand the reasoning behind the mathematical processes that use

Content

1. *Arithmetic*: Students understand how the familiar properties of division, remainders, factorization, and primes in the integers carry over to the more generalized algebraic structure of a ring

2. *Congruence*: Students understand congruence, congruence classes, modular arithmetic, and the structure of $\mathbb{Z}_p$ when p is prime

3. *Algebraic Structures*: Student will understand the definition and basic examples of a ring. They should understand the basic properties of rings and the mappings between rings including isomorphisms and homomorphisms. More specifically, students should be able to do the following as a result of the course:

    - Define a ring and identify whether a given set of elements satisfies the critera to be a ring
    - Define and give examples of the following: subring, identity, unit, inverse, field, integral domain, zero divisor, idempotent
    - Define a ring homomorphism, identify if a given mapping is a ring homomorphism, create a mapping that is a ring homomorphism
    - Define a ring isomorphism, identify if a given mapping is a ring isomorphism, create a mapping that is a ring isomorphism
    - Understand conceptually the ring structure preserved in a ring homomorphism and in a ring isomorphism

Prerequisites

Students in this course should be prepared for an upper-level mathematics course by having completed the following courses below:

1. Calculus 1
2. Calculus 2
3. Foundation of Mathematics/Discrete Mathematics/Mathematical Logic
4. Multivariable Calculus
5. *Linear Algebra

For requirement 3 above, any course that has formally introduced logic and proof will be sufficient, I have merely listed three of the most popular courses. These course names vary by institution but are characterized by their introduction to logic, proof techniques, set theory, functions, relations, induction, cardinal numbers, and number theory. Requirements 5 has a star as this could be taken concurrently with this course or not have been taken at all. The text being used does not assume any knowledge of linear algebra which is often required for an abstract algebra course. However, additional exposure to abstract concepts and proofs is extremely helpful and thus linear algebra is a preferred prerequisite but only the first four are required.

A student beginning freshman year with Calclus 1 would typically take this course as a junior. I would be open to talking to students on a case by case basis if taking the course is a good fit given their mathematical background. Aside from this background, there are also important considerations on the personal characteristics of the students taking this course. As an advanced pure math class offered in an online format, I would also strongly encourage the following personal characteristics.

1. Students should feel comfortable reading mathematical text.
2. Students should feel comfortable learning independently.
3. Student should have good time management skills.
4. Students should feel comfortable asking for help.
5. Students should have the resiliency to work through difficult problems.
6. Students should be familiar with LaTeX.
7. Students should be comfortable using ICON powered by Canvas and have access to the internet and a computer or similar device.

Reflections on Course Design

Content: This course will cover the first three chapters of Thomas Hungerford's Introduction to Abstract Algebra textbook. There are 9 sections, three in each chapter. The first six sections, Chapters 1 and 2, would be taught during the first four weeks, going at a pace of a section and a half per week. Chapter 3 would be taught during the final four weeks with the three sections in that chapter being taught at a slower pace. The e-Book is available for purchase for $65.00 on Amazon and is cheaper than that if it is rented. Since we are only covering three chapters I feel like it might be okay to provide these chapters myself. The solution manual to this book is also widely available in PDF form onlin

Format: For this course I think it makes the most sense to have at least one instructional video available for each section for 9 total videos. For original videos, I will use Doceri for iPad App and edit those videos with iMovie. There will be short discussion posts required at the end

of each week based on these videos that will ask very basic questions with respect to content in addition to one or more conceptual question. Homework will have to be uploaded into the system for grading at the end of each week and there will be an online quiz after each section. I believe that it might also be nice to have one synchronous meeting as a class once a week towards the end to discuss the content face to face. I would also have office hours on Zoom.

I think having proofs and concepts explained in videos will actually be helpful as it typically takes some time to understand everything that is happening and you don't have that option in a traditional classroom lecture. It will be important to have as much built-in feedback and scaffolding as possible and I will need to determine the most efficient way to do this. One idea is to have fill in the blank notes available to accompany each video with the homework exercises to be select examples from those notes and from questions that were posed but not answered during and after the video. This way I could facilitate the feeling of interaction through this asynchronous medium.

References and Notes

[1] Thomas Hungerford, *Abstract Algebra An Introduction (Second Edition)*, Thompson Learning, Inc. 1997.

[2] David S. Dummit, Richard M. Foote, *Abstract Algebra (Third Edition)*, New Jersey: John Wiley & Sons, Inc. 2004.

[3] Serge Lange, *Undergraduate Algebra (Third Edition)*, New York: Springer Science+Business Media, LLC, 2005.

4. It is expected that many exercises will come from the above texts in lieu of an online homework system like MyMathLab which is not optimized for this type of course.