Quiz 8 (20 pts.) Name: KEY

Short Answer

1.) (5 pts) Find the exact solution(s) to the following for all x.

(a) $2\sin\theta + \sqrt{3} = 0$

$2\sin\theta = -\sqrt{3}$

$\sin\theta = -\frac{\sqrt{3}}{2}$

$\theta = \frac{4\pi}{3} + 2k\pi, \quad \frac{5\pi}{3} + 2k\pi$

OR $\theta = 240^\circ + k360^\circ, \quad 300^\circ + k360^\circ$

(b) $\cos^2 x - 5\cos x = 6$

$\cos^2 x - 5\cos x - 6 = 0$

let $y = \cos x$

$\Rightarrow y^2 - 5y - 6 = 0$

$(y-6)(y+1) = 0$

$\Rightarrow y = 6 \quad y = -1$

$\Rightarrow \cos x = 6$

x DNE

$\Rightarrow \cos x = -1$

$x = \pi + 2k\pi$

or

$x = 180^\circ + k360^\circ$

2.) (4 pts.) Find the exact value of $\tan\left(\sec^{-1}\left(\frac{5}{3}\right) + \tan^{-1}\left(\frac{1}{3}\right)\right)$

$x + y$

$\tan(x+y) = \frac{\tan x + \tan y}{1 - \tan x \tan y}$

$\tan\left(\sec^{-1}\left(\frac{5}{3}\right) + \tan^{-1}\left(\frac{1}{3}\right)\right)$

$= \frac{\tan\left(\sec^{-1}\left(\frac{5}{3}\right)\right) + \tan\left(\tan^{-1}\left(\frac{1}{3}\right)\right)}{1 - \tan\left(\sec^{-1}\left(\frac{5}{3}\right)\right)\tan\left(\tan^{-1}\left(\frac{1}{3}\right)\right)}$

$= \frac{\left(\frac{4}{3}\right) + \left(\frac{1}{3}\right)}{1 - \left(\frac{4}{3}\right)\left(\frac{1}{3}\right)}$

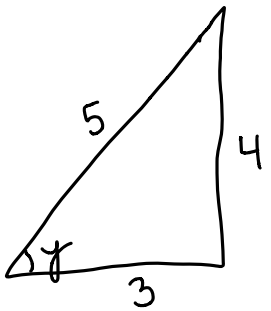
$= \frac{\left(\frac{5}{3}\right)}{\left(\frac{9}{9} - \frac{4}{9}\right)}$

$= \frac{\left(\frac{5}{3}\right)}{\left(\frac{5}{9}\right)}$

$= \frac{5}{3} \cdot \frac{9}{5}$

$= \frac{9}{3}$

$= 3$



$\sec^{-1}\left(\frac{5}{3}\right) = y$

$\Rightarrow \sec y = \frac{5}{3}$

$\Rightarrow \tan y = \frac{4}{3}$

3.) (4 pts.) Let a triangle be given with $a = 26$ ft, $b = 62$ ft, and $\alpha = 23°$. Answer the following. (*Note: round your answer to parts (a) and (b) the nearest integer*)

(a) Find β.

(b) Give the angle measurements for each possible triangle based on your choice(s) of β above.

(a) $\frac{\sin\beta}{62} = \frac{\sin(23°)}{26}$

$\sin\beta = \frac{62\sin(23°)}{26}$

$\beta = \sin^{-1}\left(\frac{62\sin(23°)}{26}\right)$

$= 68.7082$

$\approx \boxed{69°}$

$\sin(\beta) = \sin(180° - \beta)$

$\beta = 180° - 69° = \boxed{111°}$

(b)

Triangle 1	Triangle 2
$\alpha = 23°$	$\alpha = 23°$
$\beta = 69°$	$\beta = 111°$
$\gamma = 88°$	$\gamma = 46°$

4.) (4 pts.) Find the length of c in the triangle given $b = 11$ m, $\alpha = 56°$, and $\beta = 112°$.

Find γ

$\gamma = 180° - (\alpha + \beta)$

$= 180° - (56° + 112°)$

$= 180° - 168°$

$= 12°$

Find c

$\frac{\sin\gamma}{c} = \frac{\sin\beta}{b}$

$\frac{\sin(12°)}{c} = \frac{\sin(112°)}{11}$

$c = \frac{11\sin(12°)}{\sin(112°)}$

$c = 2.46664$

$\approx \boxed{2\text{ m}}$

Multiple Choice (1 pt. each)

5.) Which of the following triangles can be solved using the Law of Sines?

(a) $\alpha = 39°$, $\beta = 63°$, $\gamma = 78°$ AAA ✗

(b) $b = 139$ yd, $a = 17$ yd, $\gamma = 42°$ SSA (no side opp angle) ✗

(c) $\alpha = 105°$, $\gamma = 72°$, $c = 15$ mm AAS ✓

(d) $b = 7$ in, $a = 5$ in, $\alpha = 31°$ SSA (side opp angle) ✓

6.) Which of the following equations represents the following graph?

(a) $y = \sin^{-1} x$

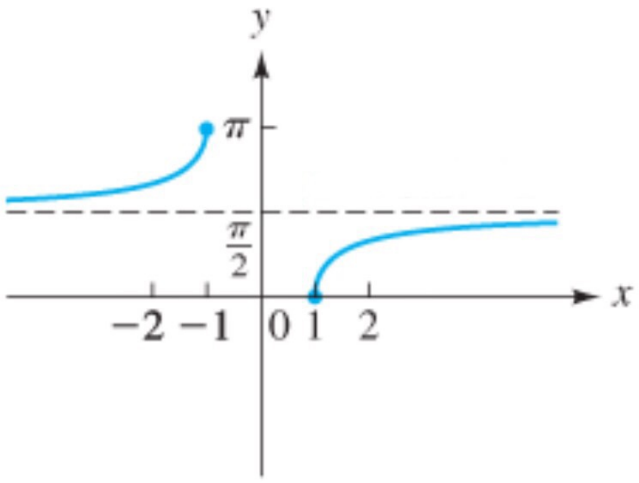
(b) $y = \cos^{-1} x$

(c) $y = \tan^{-1} x$

(d) $y = \csc^{-1} x$

(e) $y = \sec^{-1} x$

(f) $y = \cot^{-1} x$



7.) Evaluate $\theta = \csc^{-1}(2)$.

(a) $\theta = 0°$

(b) $\theta = 30°$ $\csc^{-1}(2) = \sin^{-1}\left(\frac{1}{2}\right) = 30°$

(c) $\theta = 45°$

(d) $\theta = 60°$

(e) $\theta = 90°$