NUMBERS IN REDISTRICTING

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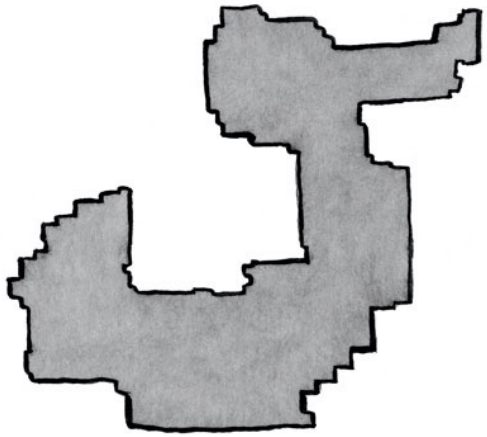
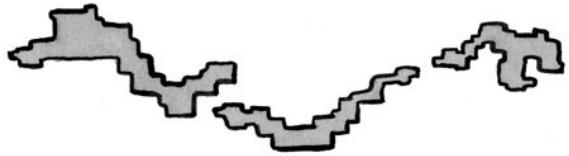
STEAM Exchange

NAME THAT GERRYMANDERED DISTRICT!



THE DUCK



THE SNAKE ON THE LAKE

REDISTRICTING

- Redistricting is the process by which new congressional and state legislative district boundaries are drawn

↳ elections are conducted in each district to select one or more representatives

↳ district lines are drawn every 10 years following the US Census

↳ seats are apportioned to states in proportion to their population

Ohio's 15 congressional districts

REDISTRICTING

Federal Requirements

Population balance: Districts should have very close to the same population

Voting Rights Act compliance: Districts cannot block minority groups from electing candidates of choice

Common State and Local Requirements

Communities of Interest: Groups with significant shared interests should be kept together

Contiguity: Each district should be one connected piece

Compactness: District shapes should be "reasonable"

Boundary preservation: District lines should follow natural and official boundaries, such as rivers or town and county borders

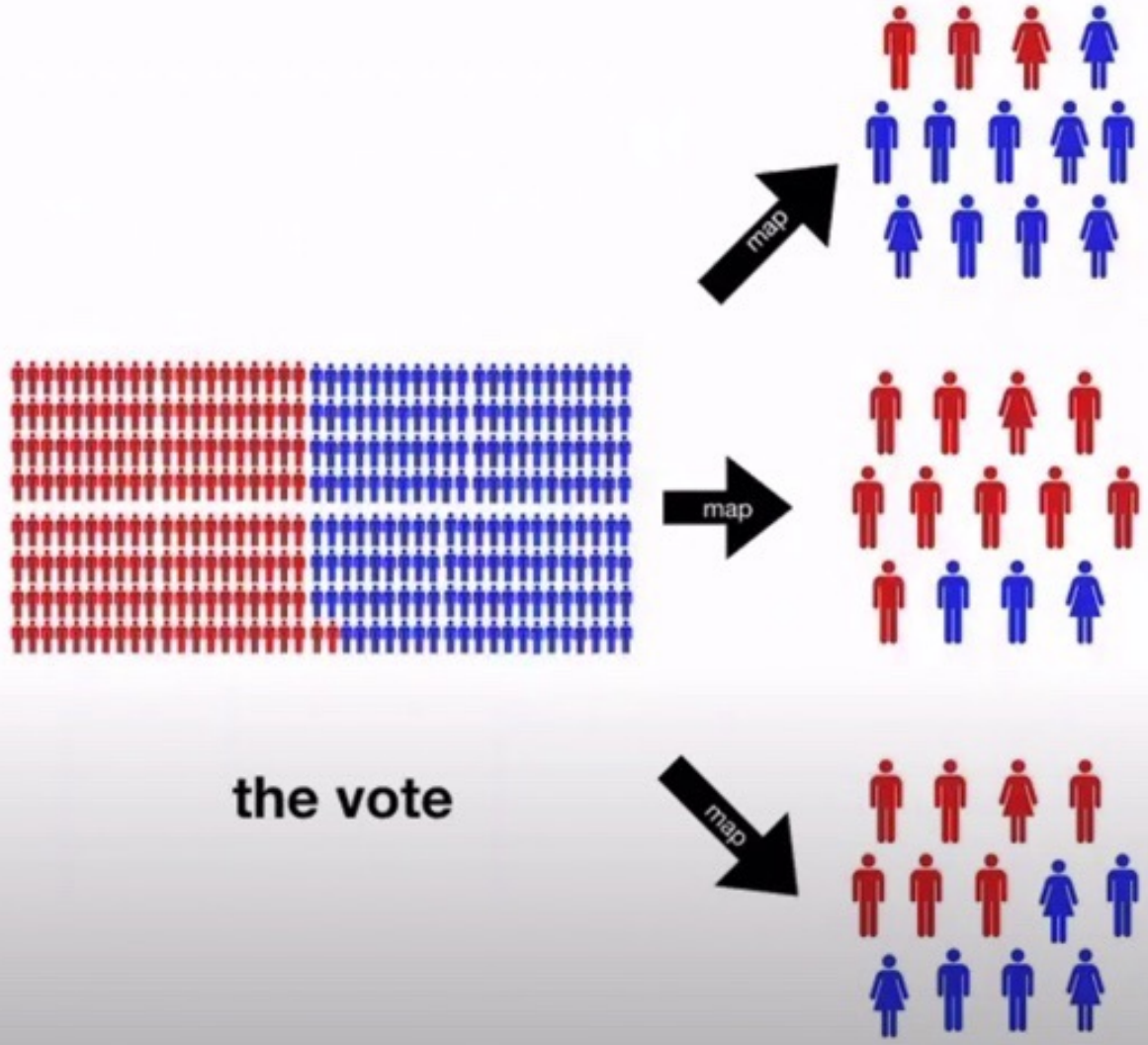
REDISTRICTING

- gerrymandering occurs when district lines are drawn to maximize the representation of one party (partisan) or class (racial) given anticipated vote patterns

- cracking - split groups across multiple districts

- packing - pack groups into as few districts as possible

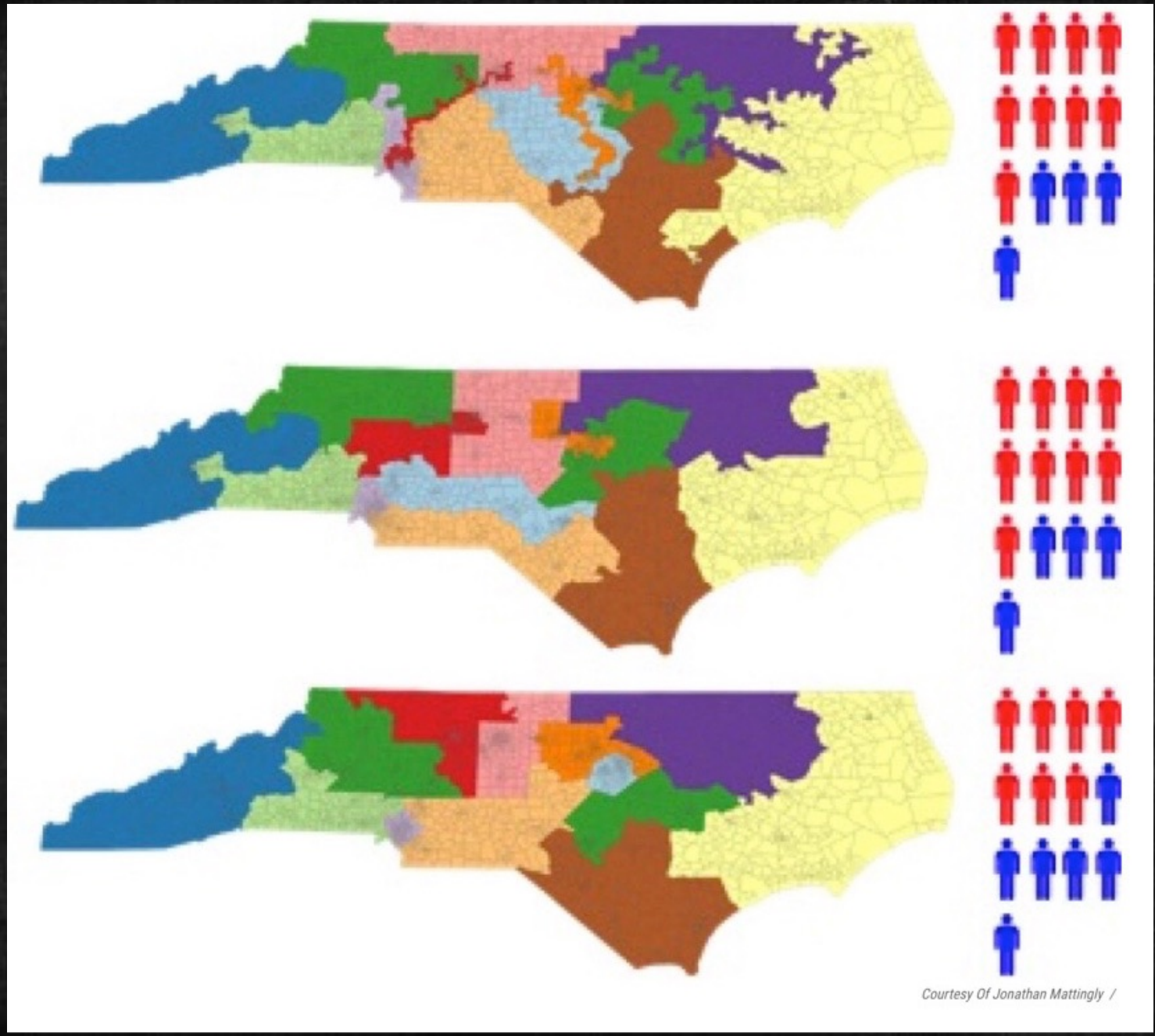
Boston Gazette March 1812



Different district maps give divergent results using the **same votes**

Choice of map determines our interpretation of the votes

Which Map ?


Courtesy Of Jonathan Mattingly /

QUANTIFYING FAIRNESS

- proportionality
  - ↳ % of votes ⟶ % of seats

- symmetry
  - ↳ same # of seats for same % of votes

- efficiency gap
  - ↳ similar # of wasted votes

COMMUNICATION

Courts, Commissions, and Consultations: How Mathematicians Are Working to End Gerrymandering

Scott Hershberger

When he entered the courtroom on October 16, 2017, mathematician Jonathan Mattingly expected to watch the proceedings all morning before being called to give testimony. He would be the fourth or fifth witness for Common Cause, an advocacy group working to end gerrymandering. But the case was on a tight four-day timeline—one day each for the plaintiffs, Common Cause and the League of Women Voters, and two days for the defendants, North Carolina redistricting officials. Right away, the three-judge panel announced that there was no disagreement about the facts, so they wanted to jump straight into expert testimony.

"[The lawyers] came over to me and said, 'Okay, Mattingly, you're on in 10 minutes.' So I was the first witness of the case, having never ever done this before, and having never been in a courtroom in my life," Mattingly recalls. "I used to watch *Perry Mason* a lot as a kid, but that was about it."

Mattingly's appearance in federal district court for *Common Cause v. Rucho* was the highest-stakes mathematics lecture of his life to that point. His goal: to convince the judges that North Carolina's 2016 congressional map was a partisan gerrymander and that mathematics offered the tools to quantify such maps.

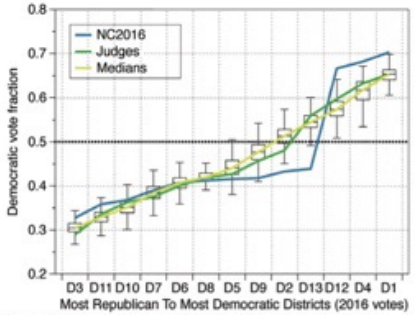


Figure 1. A version of this figure, showing the S-shape that Mattingly refers to as the "signature" of gerrymandering, appeared in the district court decision in *Common Cause v. Rucho*. Democratic votes are packed into Districts 12, 4, and 1, thus making Republicans more likely to be elected in the remaining districts. The blue line is the actual North Carolina congressional map, the green line is the map drawn during the "Beyond Gerrymandering" event at Duke, the box plots represent Mattingly's ensemble of more than 24,000 maps, and the yellow line represents the medians of the ensemble.

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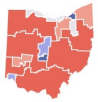
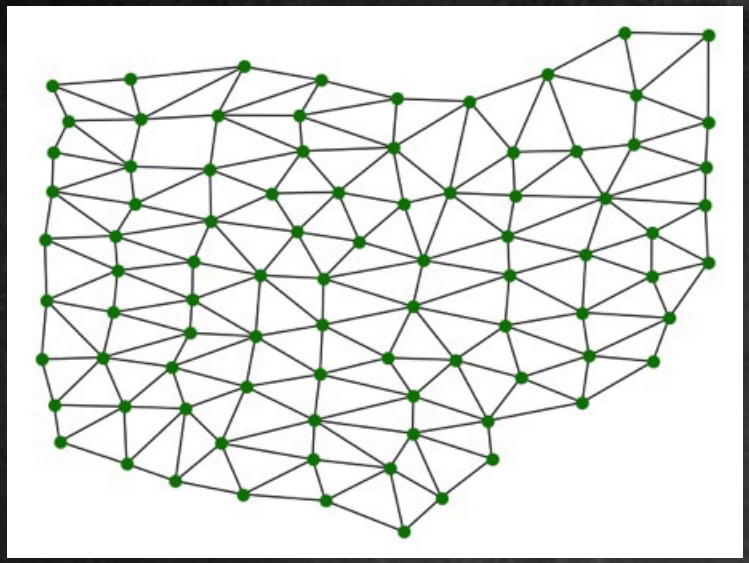
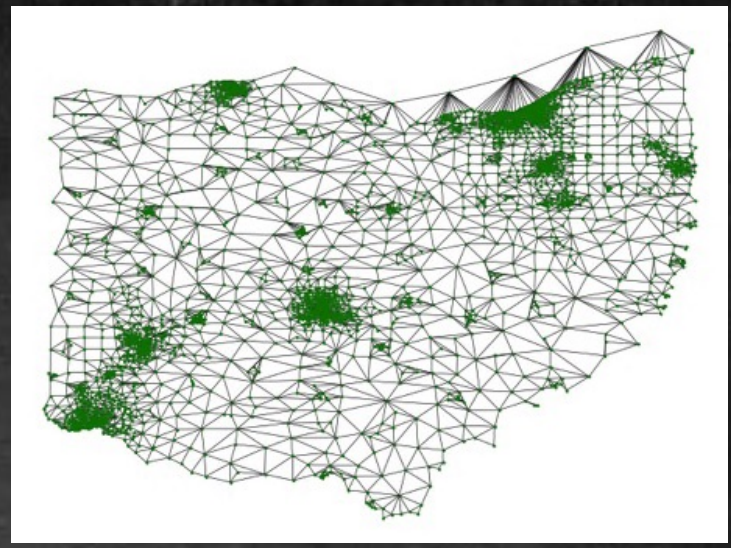
FiveThirtyEight

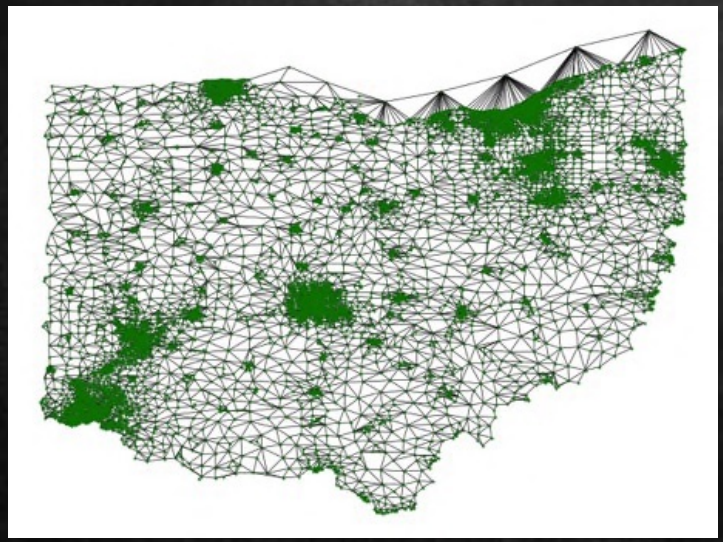
MAP	PLAN	PARTISAN BREAKDOWN
	Democratic-amended proposal	
	Republican proposal	
	Revised Democratic plan	
	Previously enacted proposal	
	State House Democratic Caucus plan	
	State Senate Republican plan	
	State House Republican plan	
	State Senate Democratic Caucus plan	



County
88

census tract
2,952

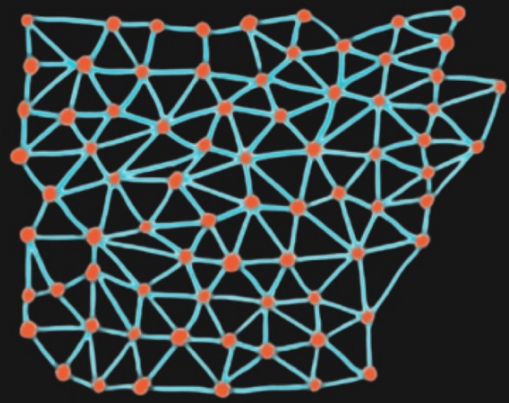


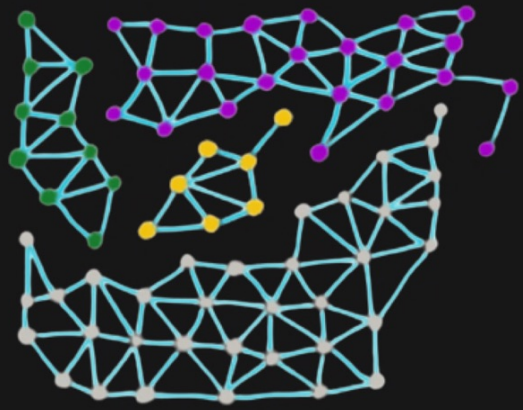


Census block group
9,238

census block
365,344

REDISTRICTING





state

graph

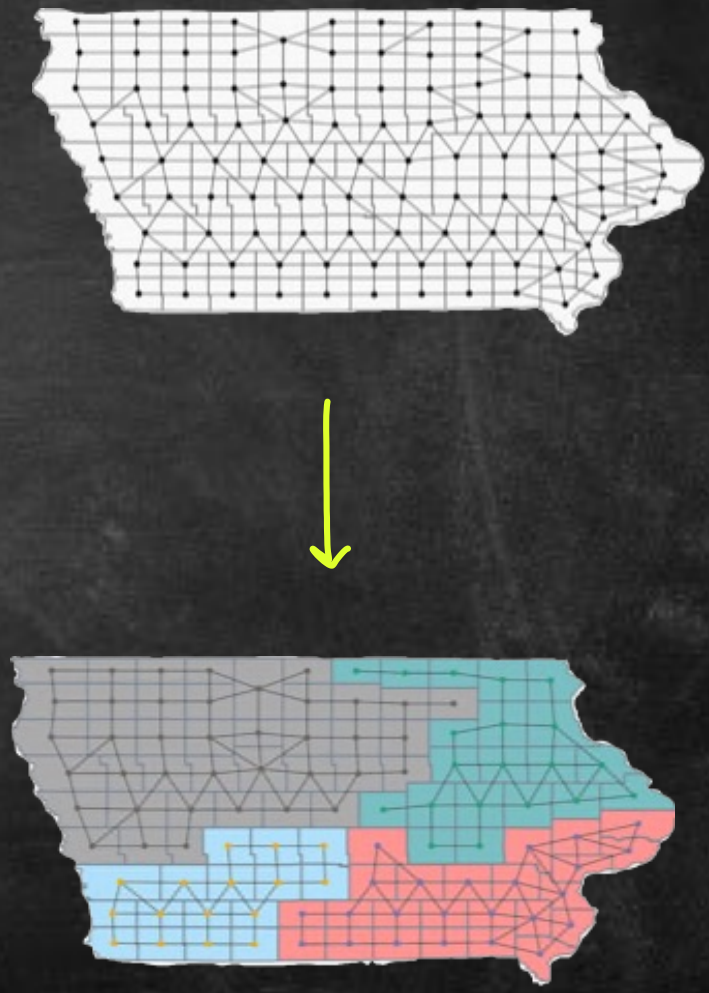
graph partition

REDISTRICTING

Let $G = (V, E)$ (connected, planar graph)

- a k-partition $P = (V_1, \dots, V_k)$ of G is a collection of disjoint subsets $V_i \subseteq V$ such that $V_1 \sqcup \cdots \sqcup V_k = V$
- $\mathcal{P}_k(G)$ denotes the collection of all k-partitions of G
- $P_i = (V_i, E_i)$ induced graphs on $V = V_1 \sqcup \cdots \sqcup V_k$ with each P_i connected

Each $P \in \mathcal{P}_k(G)$ is a districting plan, and each P_i is a district



REDISTRICTING

Given geography (G) at a particular scale, we want a partition $P \in P_k(G)$ that satisfies certain constraints:

Constraints (federal and/or state rules)

- contiguity
- population balance
- compactness
- Voting Rights Act
- municipal boundaries
- communities of interest

We want to operationalize these rules; i.e. develop mathematical formulations appropriate for this context

CONSTRAINTS

Let $G = (V, E)$ be given and consider $\mathcal{P}_k(G)$

contiguity

☑ each induced graph $P_i = (V_i, E_i)$ on $V = V_1 \sqcup \cdots \sqcup V_k$ is connected

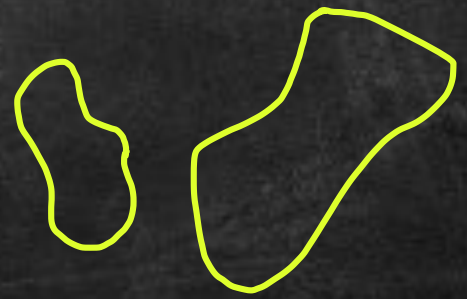
Contiguous

population balance

- $\rho : V_i \longrightarrow \mathbb{R}$ population of district P_i
- $p(G) = \sum_i p(V_i)$

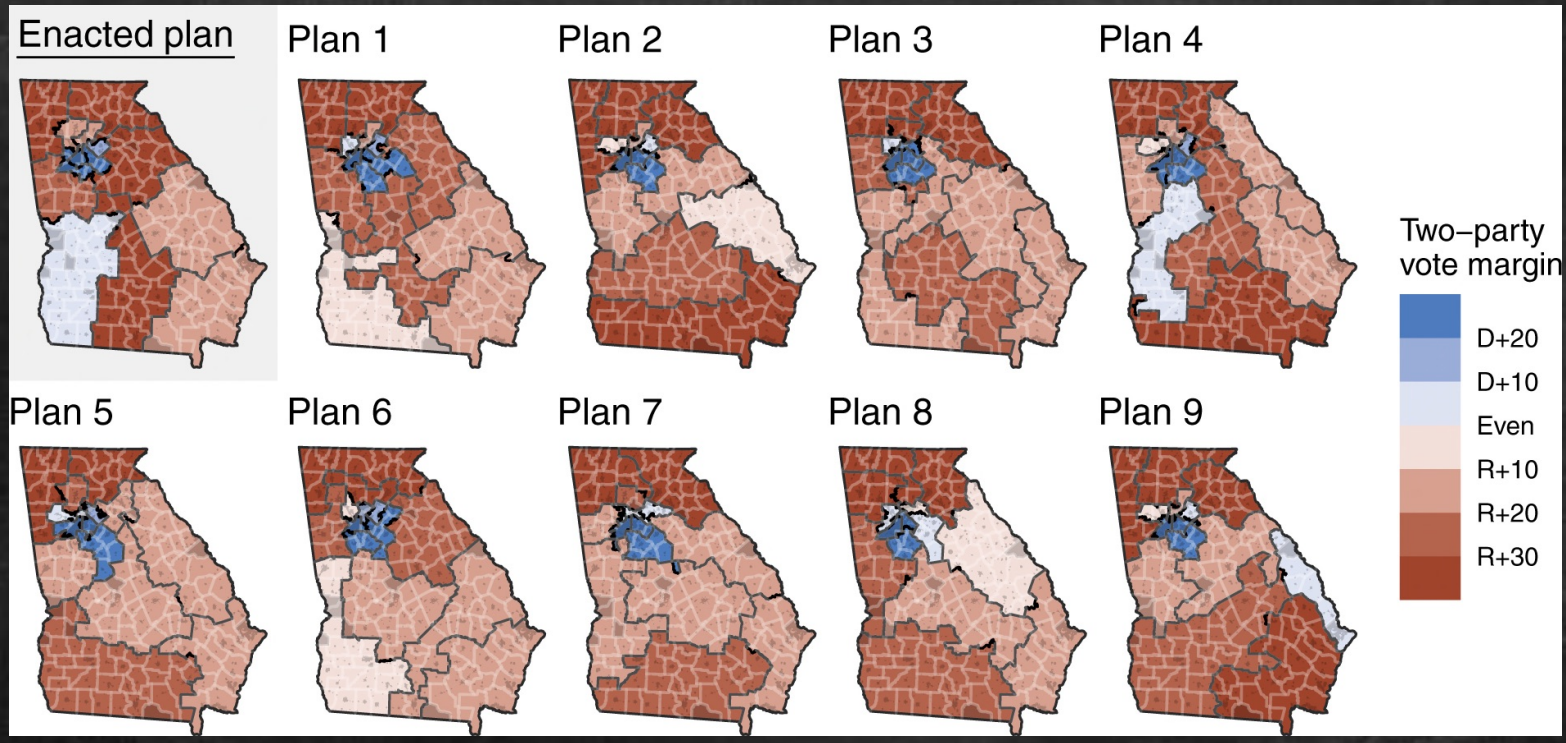
☑ $(1-\varepsilon) \cdot \frac{p(G)}{k} \leq p(P_i) \leq (1+\varepsilon) \cdot \frac{p(G)}{k}$

for all i



not contiguous

SPACE OF PLANS



Can we enumerate all plans and pick the best one?

ENSEMBLE ANALYSIS

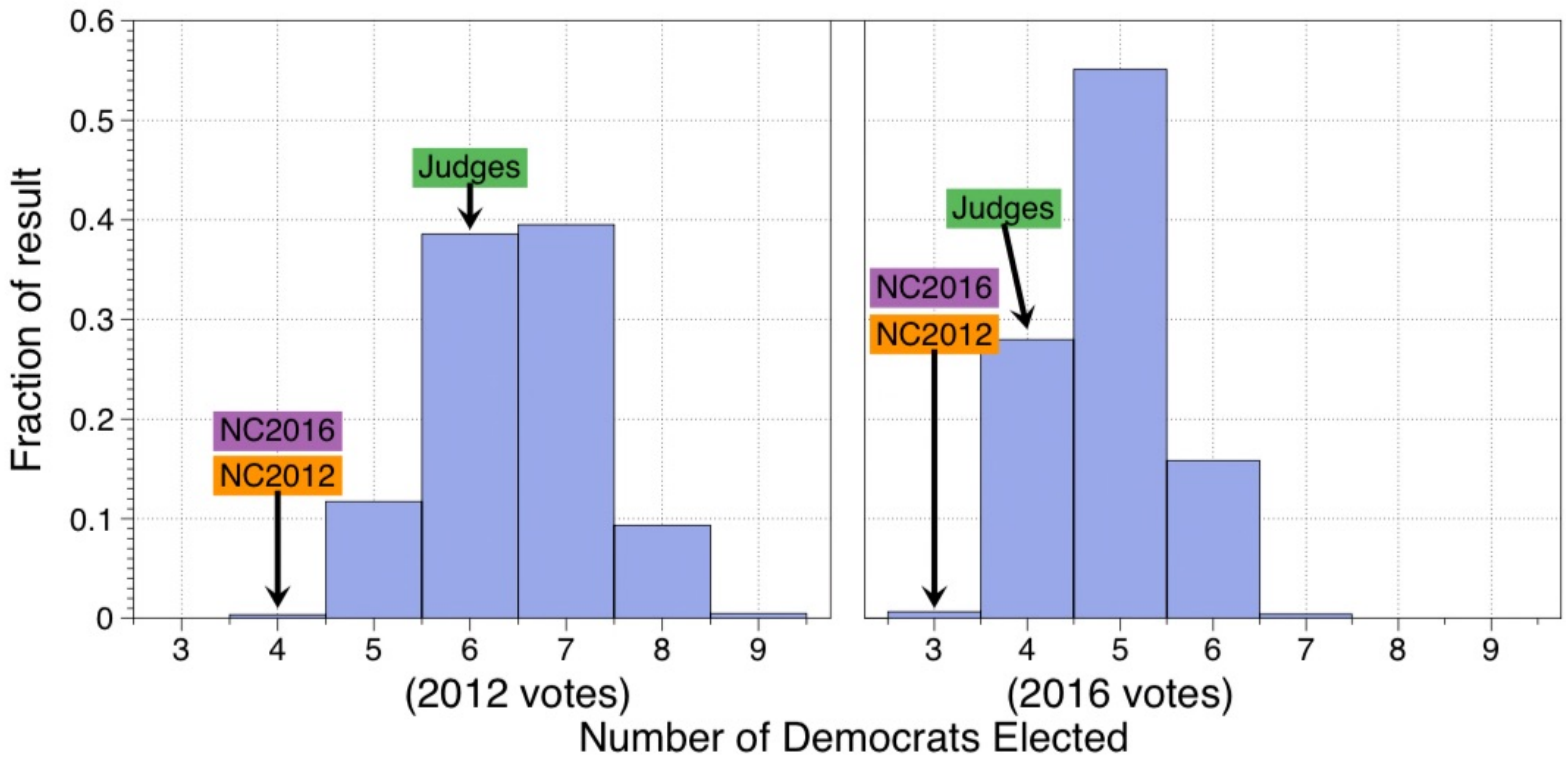


Fig. 1. Probability of a given number of Democratics elected among the 13 congressional seats using votes from the 2012 election (left) and 2016 election (right).

ENSEMBLE ANALYSIS

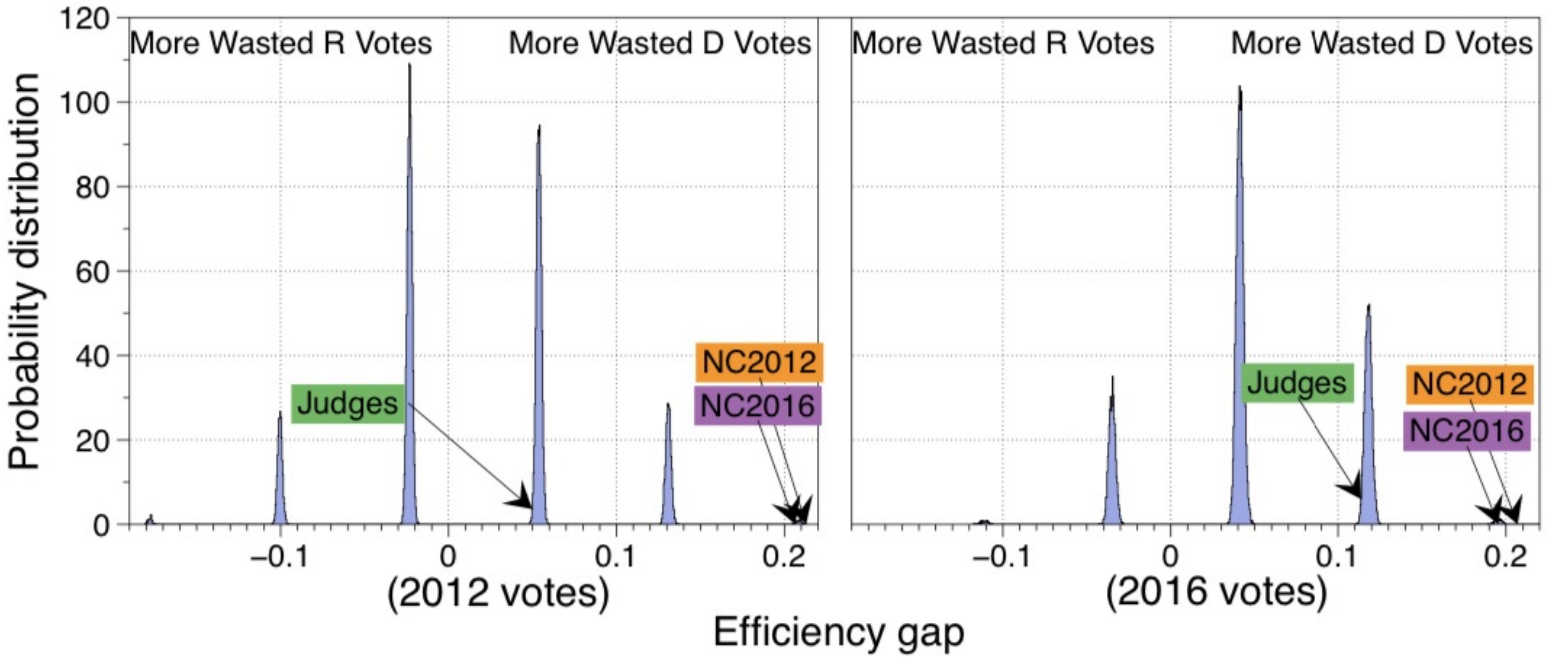


Fig. 4. Absolute values of partisan bias (top) and the efficiency gap (bottom) for the three districts of interest based on the voting data from 2012 (left) and 2016 (right).

LEGAL BATTLES

- Rucho vs. Common Cause (PA 2016 map)
  - ↳ mathematician submitted amicus brief
  - ↳ "partisan gerrymandering is nonjusticiable"

- League of Women Voters of Pennsylvania vs. Commonwealth of Pennsylvania
  - ↳ invalidated 2011 PA congressional map

- League of Women Voters vs. Ohio Redistricting Commission
  - ↳ invalidated initial 2021 OH congressional map

QUANTIFYING FAIRNESS

Takeaways:

- "Gerrymanders" are statistical outliers in an ensemble of valid redistricting plans

Theme 1: Operationalizing the Rules

- How do we quantify rules and priorities that govern the redistricting process?

Theme 2: Space of Plans

- How do we sample efficiently from the space of plans?
- How do we compare $P, P' \in \mathcal{P}_k(G)$

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ENSEMBLE ANALYSIS

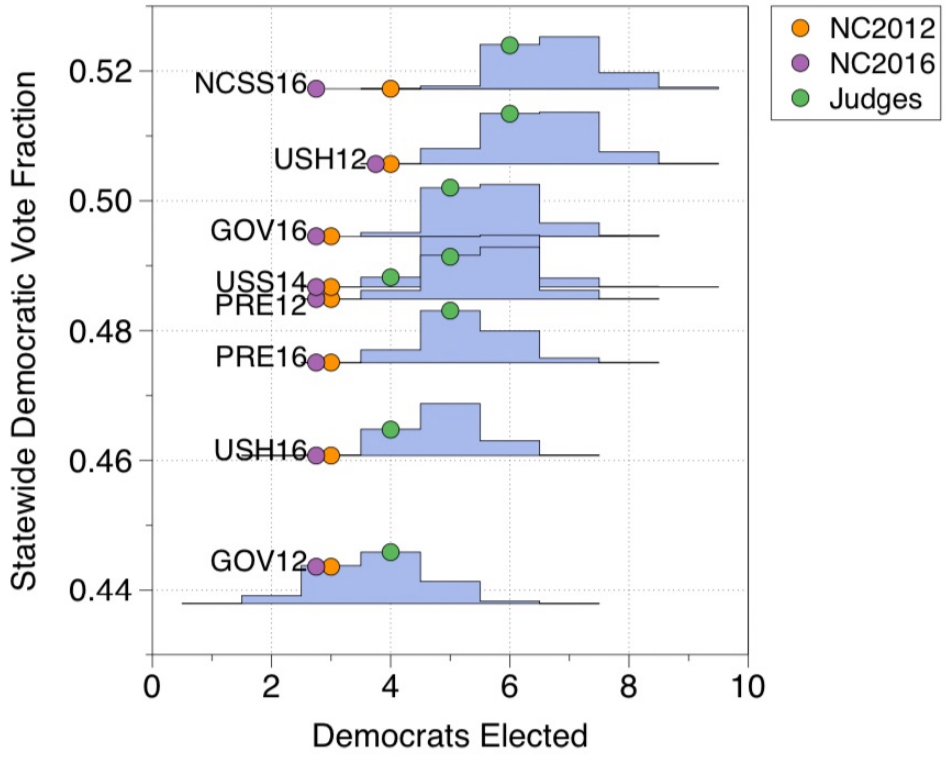


Fig. 2. Probability of a given number of Democratic wins among the 13 congressional seats using vote counts from a variety of elections. The y-axis shows the statewide democratic vote fraction. Elections shown are the 2012 and 2016 presidential races (PRE12, PRE16), the 2016 North Carolina secretary of state race (NCSS16), the 2012 and 2016 gubernatorial races (GOV12, GOV16), the 2014 and 2016 US senatorial races (USS14), and the 2012 and 2016 US congressional races (USH12, USH16; also shown in Figure 1).

ENSEMBLE ANALYSIS

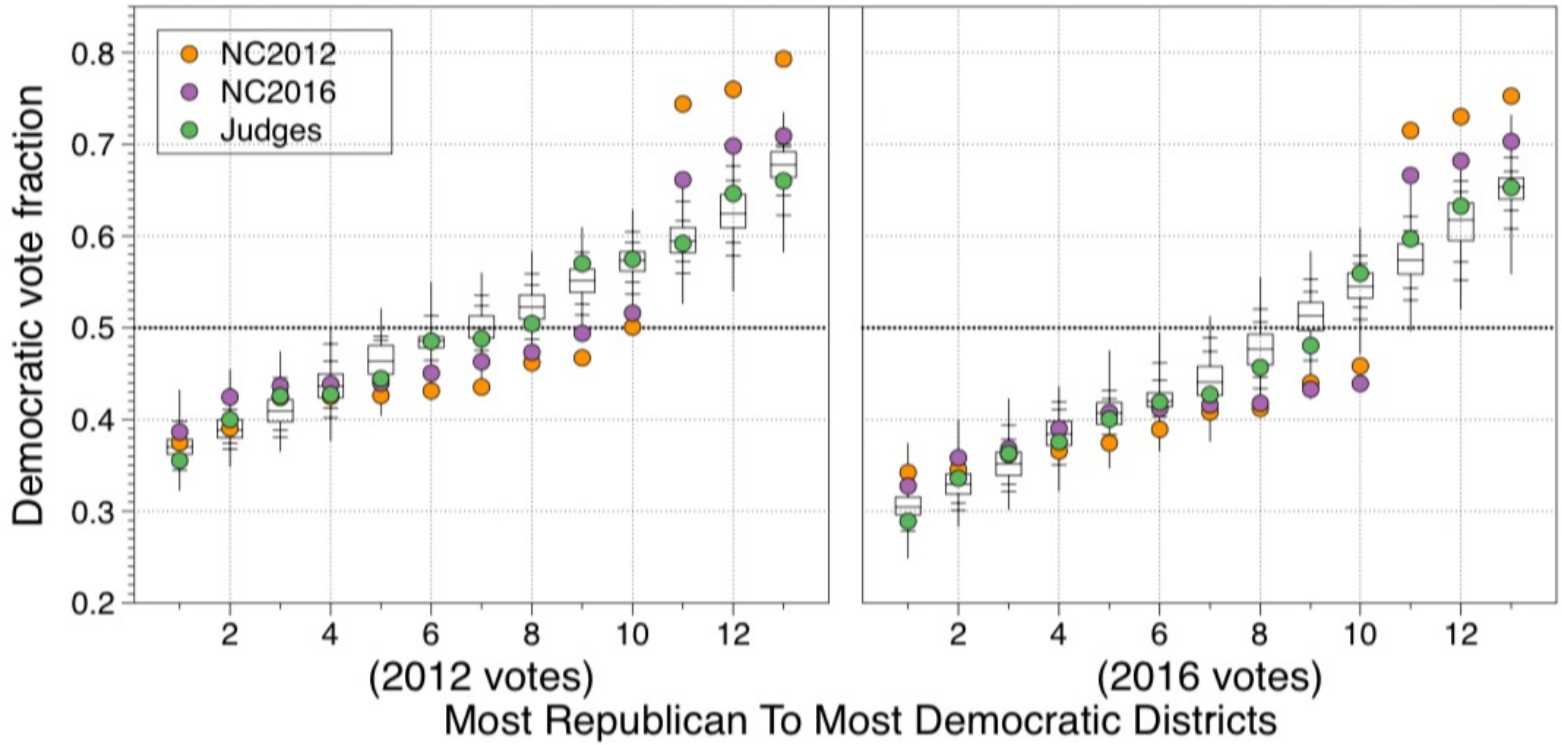


Fig. 3. Box-plot summary of districts ordered from most Republican to most Democratic, for the voting data from 2012 (left) and 2016 (right). We compare our statistical results with the three redistricting plans of interest.

OHIO CITIZENS REDISTRICTING COMMISSION

OFFICIAL REPORT TO THE OHIO REDISTRICTING COMMISSION

AUGUST 2021

E-Mail: info@commissionocrc.org
Web: www.ohredistrict.org

Community Mapping Project

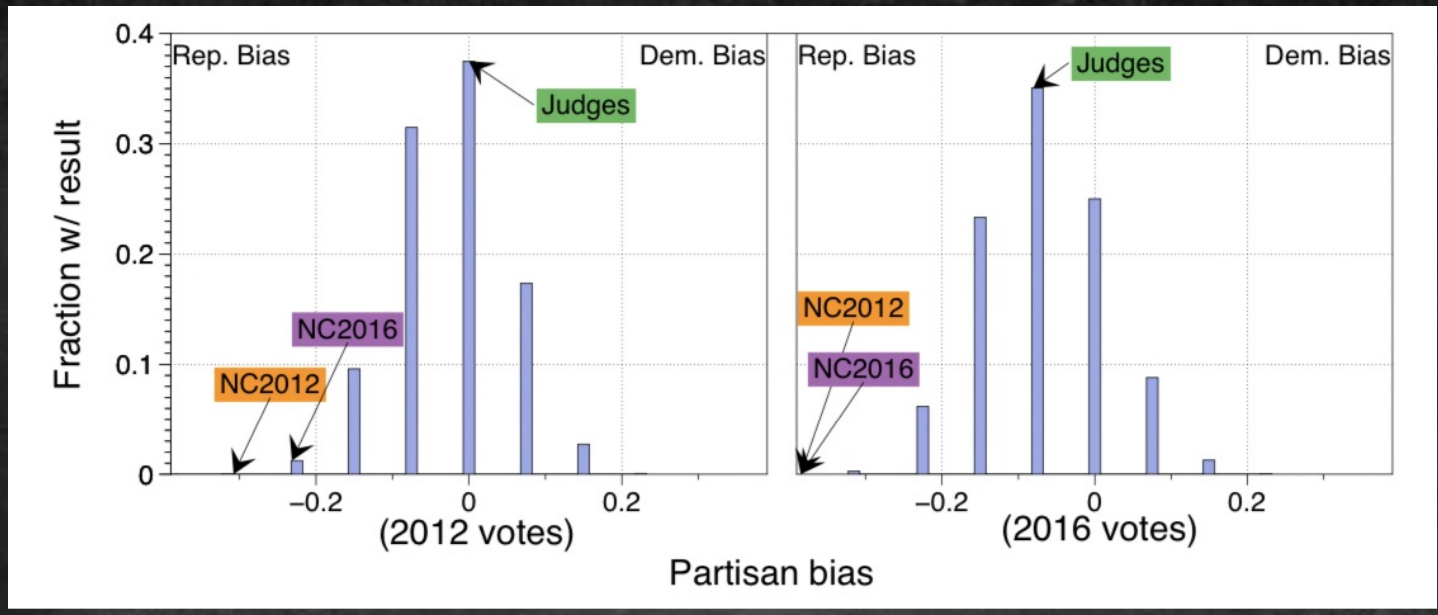
The MGGG Redistricting Lab built a project team based at The Ohio State University and supported by a network of grassroots organizations to collect and synthesize Community of Interest (COI) input for the OCRC. One of the major guiding principles of this team was to ensure that narratives, needs, and concerns from a diverse range of Ohioans were included in the process. They emphasized concerted outreach to minority and underrepresented communities in their approach.

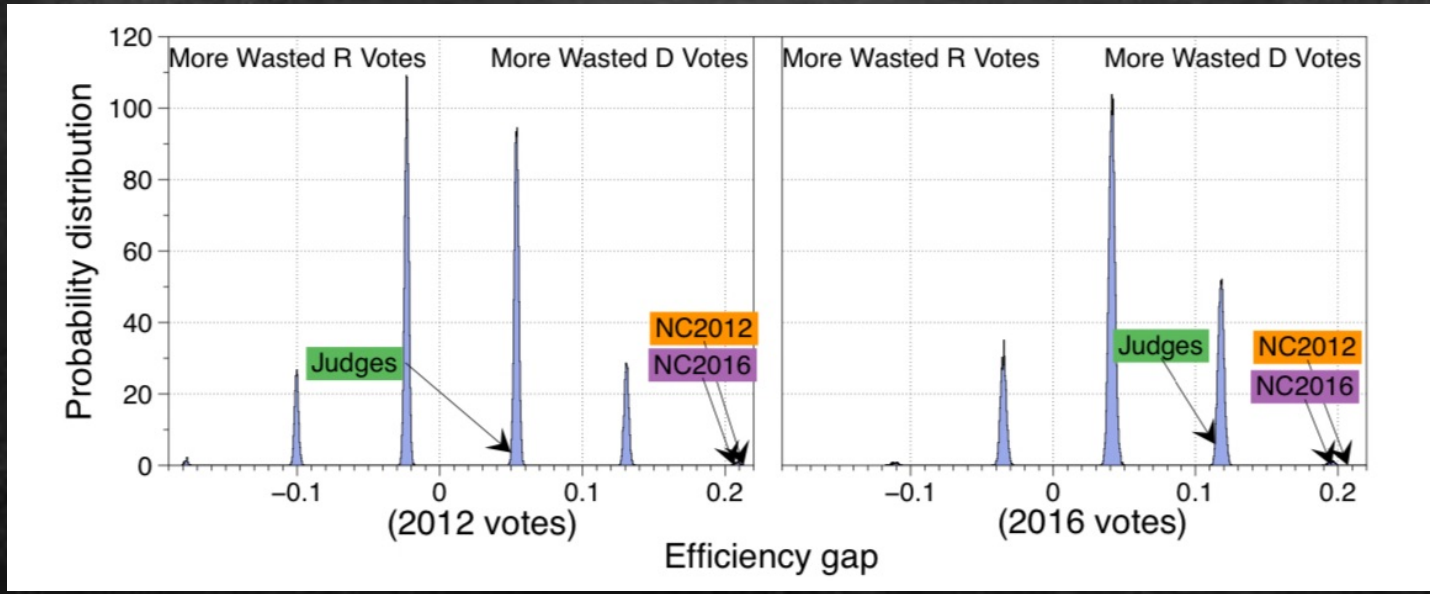
This team used Districtr, a free community web tool developed by MGGG to enable users to create both COI regions and "points of interest" paired with narratives about community issues and needs. **There were 2,350 submissions received through the Districtr portal.**

Prioritizing Communities of Interest is generally considered to be essential to drawing fair districts, but in practice, it is prohibitively difficult to implement without local community knowledge. This community mapping project collected spatialized testimony from the public, which featured not only narrative descriptions of the communities, but mapping describing their geography.

Districtr users could work remotely or join video conference-based workshops led by members of our outreach partner organizations. The Commission received public input in many modalities: collection at in-person meetings when possible, in virtual public meetings, submitted through Districtr.org, OCRC website and email or even via social media.

OCRC Districtr Community of Interest heat map clusters from East Columbus, Reynoldsburg, Gahanna and Westerville prepared by MGGG.


Rep. Bias
Dem. Bias
Judges
NC2016
NC2012
Fraction w/ result
0.4
0.3
0.2
0.1
0
−0.2
0
0.2
(2012 votes)
Rep. Bias
Judges
Dem. Bias
NC2012
NC2016
−0.2
0
0.2
(2016 votes)
Partisan bias


More Wasted R Votes
More Wasted D Votes
Judges
NC2012
NC2016
Probability distribution
120
100
80
60
40
20
0
−0.1
0
0.1
0.2
(2012 votes)
More Wasted R Votes
More Wasted D Votes
Judges
NC2012
NC2016
−0.1
0
0.1
0.2
(2016 votes)
Efficiency gap