Take Home Quiz 6 (20 pts.) Name: KEY

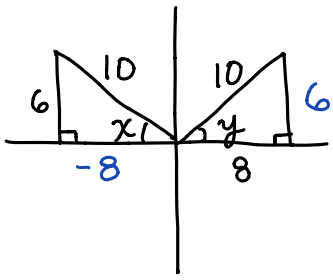
Short Answer

1.) (4 pts.) Prove $\tan 2x = \dfrac{2\tan x}{1-\tan^2 x}$ using the sum identity for $\tan(x+y)$.

$\boxed{\tan(x+y) = \dfrac{\tan x + \tan y}{1 - \tan x \tan y}}$

$$\tan(2x) = \tan(x+x)$$
$$= \frac{\tan x + \tan x}{1 - \tan x \tan x}$$
$$= \frac{2\tan x}{1 - \tan^2 x}$$

2.) (4 pts.) Find the exact value of $\cos(x+y)$ given $\sin x = \frac{6}{10}, \cos y = \frac{8}{10}$, x is in Quadrant II, and y is in quadrant I.



$\boxed{\cos(x+y) = \cos x \cos y - \sin x \sin y}$

$$\cos(x+y) = \left(\frac{-8}{10}\right)\left(\frac{8}{10}\right) - \left(\frac{6}{10}\right)\left(\frac{6}{10}\right)$$
$$= \frac{64}{100} - \frac{36}{100}$$
$$= \frac{28}{100} \div \frac{4}{4}$$
$$= \frac{7}{25}$$

3.) (4 pts.) Find the exact value of $\tan(135°)$.

$$\tan(x-y) = \frac{\tan x - \tan y}{1+\tan x \tan y}$$

$$\begin{aligned}\tan(135°) &= \tan(180°-45°)\\ &= \frac{\tan(180°)-\tan(45°)}{1+\tan(180°)\tan(45°)}\\ &= \frac{(0)-(1)}{1+(0)(1)}\\ &= \frac{-1}{1}\\ &= -1\end{aligned}$$

4.) (4 pts.) Verify the following identities:

(a) $\dfrac{1-(\sin\theta-\cos\theta)^2}{\sin\theta} = 2\cos\theta$

$$\begin{aligned}LHS &= \frac{1-[\sin^2\theta - 2\sin\theta\cos\theta + \cos^2\theta]}{\sin\theta}\\ &= \frac{1-[1-2\sin\theta\cos\theta]}{\sin\theta}\\ &= \frac{1-1+2\sin\theta\cos\theta}{\sin\theta}\\ &= \frac{2\sin\theta\cos\theta}{\sin\theta}\\ &= 2\cos\theta\end{aligned}$$

(b) $\csc^2\theta - \cos^2\theta - \sin^2\theta = \cot^2\theta$

$$\begin{aligned}\csc^2\theta - [\cos^2\theta+\sin^2\theta] &= \csc^2\theta - 1\\ &= \cot^2\theta\end{aligned}$$

$$\frac{\sin^2\theta}{\sin^2\theta} + \frac{\cos^2\theta}{\sin^2\theta} = \frac{1}{\sin^2\theta}$$

$$1+\cot^2\theta = \csc^2\theta$$

5.) (4 pts.) Is $\sin(2\pi - x) = \sin(-x)$ an identity?

$$\sin(x-y) = \sin x\cos y - \cos x\sin y$$

$$\begin{aligned}\sin(2\pi - x) &= \sin(2\pi)\cos(x) - \cos(2\pi)\sin x\\ &= (0)\cos x - (1)\sin x\\ &= -\sin x\\ &= \sin(-x)\end{aligned}$$

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