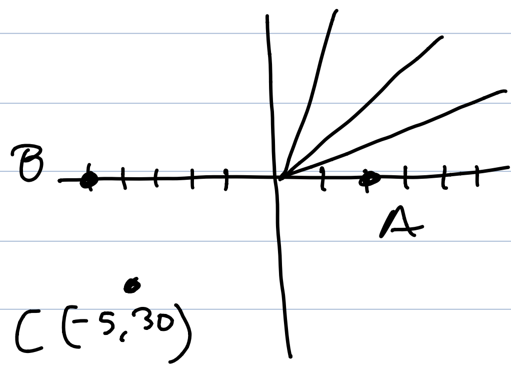
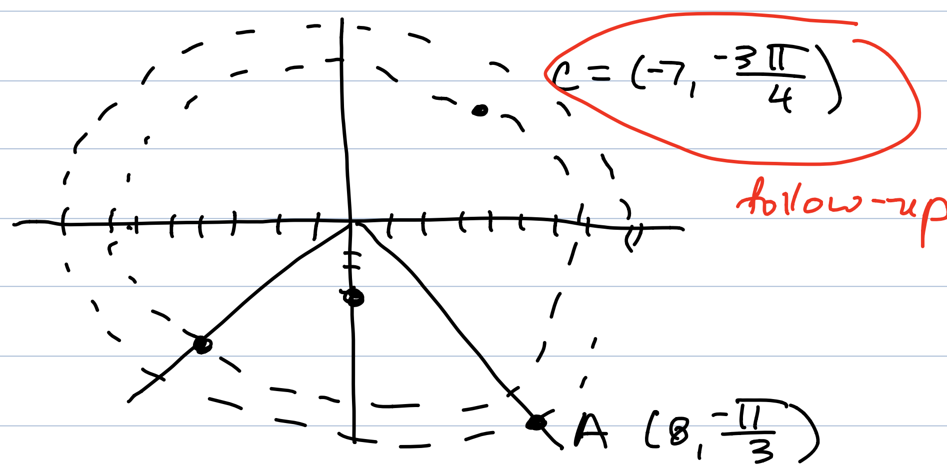
ROUND 1: plotting polar coordinates

Q1) Plot $A = (2, 0^\circ)$, $B = (5, 180^\circ)$, $C = (-5, 30^\circ)$



Q2) Plot $A = \left(8, \frac{-\pi}{3}\right)$, $B = \left(3, \frac{-\pi}{2}\right)$, $C = \left(-7, \frac{-3\pi}{4}\right)$



ROUND 2: polar / rectangular coordinates

Q1) Change $P=(5, -\pi/4)$ to rectangular coordinates

$$\boxed{P=\left(\frac{5\sqrt{2}}{2}, -\frac{5\sqrt{2}}{2}\right)}$$

$x = r\cos\theta$
$= (5)\cos(-\pi/4)$
$= 5(\sqrt{2}/2)$
$= \frac{5\sqrt{2}}{2}$

$y = r\sin\theta$
$= 5\sin(-\pi/4)$
$= 5(-\sqrt{2}/2)$
$= -\frac{5\sqrt{2}}{2}$

Q2) Change $P=(\sqrt{3}, \sqrt{3})$ to polar coordinates with $r \geq 0$ and $-\pi < \theta \leq \pi$

$$\boxed{P=\left(\sqrt{6}, \frac{\pi}{4}\right)}$$

$r^2 = (\sqrt{3})^2 + (\sqrt{3})^2 = 3+3 = 6 \Rightarrow r = \pm\sqrt{6}$ ← choose $r=\sqrt{6}$ since $r \geq 0$

$\tan\theta = \frac{\sqrt{3}}{\sqrt{3}} = 1 \Rightarrow \theta = \tan^{-1}(1) = \frac{\pi}{4}$

in Quadrant I since not III $r \geq 0$.

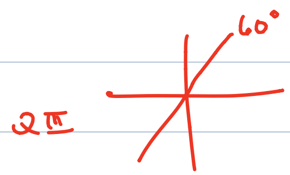
Q3) change $(-1,-\sqrt{3})$ to polar coordinates with $(r \geq 0,\ 0 \leq \theta < 2\pi)$

$$\boxed{(r, \theta) = \left(2, \frac{4\pi}{3}\right)}$$

$r^2 = (-1)^2 + (-\sqrt{3})^2 = 1+3 = 4$

$\Rightarrow r = \pm\sqrt{4}$ $\quad \underline{r = 2}$ since $r \geq 0$

$\tan\theta = \dfrac{-\sqrt{3}}{-1} = \sqrt{3} \Rightarrow \theta = \tan^{-1}(\sqrt{3}) = \dfrac{4\pi}{3}$



ROUND 3: polar/rectangular equations

Q1) Find a polar equation ($r=f(\theta)$) for the line $y=8$

$$\boxed{r=\frac{8}{\sin\theta}}$$

$y=8$

$\Rightarrow r\sin\theta=8$

$\Rightarrow r=\dfrac{8}{\sin\theta}$

Q2) Change $13y-y^2=x^2$ to polar form

$$\boxed{r=13\sin\theta}$$

$13y-y^2=x^2$

$0=x^2+y^2-13y$

$0=r^2-13r\sin\theta$

$0=r(r-13\sin\theta)$

$\Rightarrow r=0 \quad r-13\sin\theta=0$

$r=13\sin\theta$

Q3) Change $3x+7y=13$ to polar form.

$$\boxed{r=\frac{13}{3\cos\theta+7\sin\theta}}$$

$3x+7y=13$

$3r\cos\theta+7r\sin\theta=13$

$\Rightarrow r(3\cos\theta+7\sin\theta)=13$

$\Rightarrow r=\dfrac{13}{3\cos\theta+7\sin\theta}$

Q4) Change $x^2+y^2=36$ to polar form.

$\boxed{r=6\ ;\ r=-6}$

$$x^2+y^2=36$$
$$r^2=36$$
$$r=\pm\sqrt{36}$$
$$r=6\ ;\ r=-6$$

Q5) change $4\cos\theta+\sin\theta=\frac{17}{r}$ to rectangular form.

$\boxed{4x+y-17=0}$

$$4\cos\theta+\sin\theta=\frac{17}{r}$$

$$r(4\cos\theta+\sin\theta)=r\cdot\frac{17}{r}$$
$$4r\cos\theta+r\sin\theta=17$$
$$4x+y=17$$
$$4x+y-17=0$$

Q6) change $r=5\cos\theta+6\sin\theta$ to rectangular form

$\boxed{x^2+y^2-5x-6y=0}$

$$r=5\cos\theta+6\sin\theta$$
$$r^2=5r\cos\theta+6r\sin\theta$$
$$x^2+y^2=5x+6y$$
$$x^2+y^2-5x-6y=0$$

Q7) change $r^2\cos 2\theta = 8$ to rectangular form.

$\boxed{x^2 - y^2 - 8 = 0}$

$r^2\cos 2\theta = 8$

$r^2(\cos^2\theta - \sin^2\theta) = 8$

$\underbrace{r^2\cos^2\theta} - \underbrace{r^2\sin^2\theta} = 8$

$x^2 - y^2 = 8$

$x^2 - y^2 - 8 = 0$